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PROBLEM OF GAUGE INVARIANCE IN THE THEORY OF THE MEISSNER EFFECT

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This is to reply to "a note on the theory of the Meissner effect" by Pines and Schrieffer, recently published in these Letters,[1] which criticizes my paper on the Meissner effect[2] and gives preference to a different mathematical approach. Since the method recommended by them is essentially that used by Rickayzen,[3] I want to confront Rickayzen's method with mine. Both methods are based on the same, strictly gauge invariant, Hamiltonian. The phonons are treated as real particles, coupled to the electrons by a Bloch-Fröhlich interaction with a coupling strength g.

The mathematical difficulty of the present theory of superconductivity lies in the fact that certain effects of the phonon-electron coupling must be taken into account "rigorously," i.e., without expansion into powers of g, because they involve functions nonanalytic in g as $g\to 0$ (e.g., the "energy gap"). However, it is only a "reduced" problem that is solved rigorously, and "higher order" effects are still discussed in terms of an expansion in g. This is true both for Rickayzen's paper (see reference 3, Sec. 3) and mine (reference 2, Sec. 1). One hopes that, owing to the correct choice of the "reduced problem," the remaining expansion will converge rapidly if g is small. The results of the two papers are at variance because different reductions are used, and the question is which, if any, is right.

In the absence of a rigorous derivation, one necessary criterion is supplied by the gauge invariance of the result. In order to make use of this criterion it is mandatory to leave the gauge of the magnetic vector potential unrestricted, in other words to introduce *ad hoc* a longitudinal vector potential, $\vec{A}_{long}$, and show that its effects vanish identically, for the chosen reduced problem with subsequent expansion in g. Setting $\vec{A}_{long} = 0$ (or div $\vec{A} = 0$) as Rickayzen does, means foregoing the gauge invariance test (not of the Hamiltonian but of the calculational method).

Whereas my method is designed so as to meet this test, Rickayzen's method when applied to $\vec{A}_{long}$ gives a nonvanishing current $\vec{j}_{long}$ (the coefficient has the London or BCS value as $g\to 0$). One must then conclude that Rickayzen's g expansion does not converge rapidly, at least in the case of $\vec{j}_{long}$. The question remains whether there is any reason to believe that the corresponding expansion for $\vec{j}_{trans}$ is more trustworthy.

In this context, Pines and Schrieffer mention the plasma vibrations which are, indeed, excited by $\vec{A}_{long}$ but not by $\vec{A}_{trans}$.[4] However, for applying our criterion it is not necessary to introduce any Coulomb interactions at all; the latter problem is quite difficult in itself and requires other very doubtful approximations.[5] In order to make the mathematical test as clear-cut as possible it is preferable to omit the Coulomb interactions in the Hamiltonian altogether.

Even then, there exist "collective excitations,"[6] both of longitudinal and transverse type, and one may ask whether their contribution to $\vec{j}_{long}$ might tend to cancel the wrong "leading term" in Rickayzen's theory. Examples for longitudinal collective modes are phonons clothed with quasi-particle pairs, and quasi-particle pairs bound by the phonon field.[6] Applying customary approximations (e.g., Tamm-Dancoff) to these excitations, I tried to find terms which might help to restore the correct result $\vec{j}_{long} = 0$, but I did

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not succeed. I came to the conclusion that Rickayzen's method, in the Meissner effect problem, does not provide a proper starting point for subsequent low-order approximations, and that, at any rate, his power expansions in g cannot be trusted. If high-order terms contribute essentially to $\vec{j}_{long}$, they can certainly also affect $\vec{j}_{trans}$. The "leading term" may well be equally misleading in both cases.

In view of these doubts, it seemed a decisive advantage first to transform the Hamiltonian into a manifestly gauge-invariant form [reference 2, Eq. (2) with (1) and (17)]. In this new representation, the current operator $\vec{j}$ is again obtained as a power series in g [Eq. (10)], but now $\vec{j}_{long}$ vanishes automatically, to all orders in g. This does not prove, of course, that the power series for $\vec{j}_{trans}$ converges rapidly. But Pines' and Schrieffer's criticism of this expansion is groundless and futile because it is based on a comparison with a low-order approximation that is in essence the same as Rickayzen's.[7]

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[1]D. Pines and J. R. Schrieffer, Phys. Rev. Lett. 1, 407 (1958). Dr. Schrieffer kindly gave me a copy, and we discussed the subject.

[2]G. Wentzel, Phys. Rev. 111, 1488 (1958).

[3]G. Rickayzen, Phys. Rev. 111, 817 (1958).

[4]J. Bardeen, Nuovo cimento 5, 1765 (1957); P. W. Anderson, Phys. Rev. 110, 827 (1958); D. Pines and J. R. Schrieffer, Nuovo cimento (to be published).

[5]E. N. Adams, Phys. Rev. 98, 1130 (1955).

[6]Corresponding to isolated roots of a secular equation. Compare Bogoliubov, Tolmachov, and Shirkov, "A new method in the theory of superconductivity," Dubna, June, 1958, sections 3.2-4; P. W. Anderson, Phys. Rev. (to be published).

[7]To realize this, one need only follow the argumentation in reference 1 (on p. 408). Note the replacement of Ψ_α and Ψ_β by eigenfunctions of a reduced Hamiltonian, and compare with reference 3.

UNSTABLE PLASMA OSCILLATIONS IN A MAGNETIC FIELD

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This work treats the small amplitude oscillations of a fully ionized quasi-neutral plasma in a uniform time-independent externally produced magnetic field. Motions of the ions and perturbations of the magnetic field are neglected. The

distribution function $f(\vec{r},\vec{v},t)$ for the electrons satisfies

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial \vec{r}}-\frac{e}{m}\left(\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}\right)\cdot\frac{\partial f}{\partial \vec{v}}=0, \tag{1}$$

and the electric field which appears in Eq. (1) satisfies

$$\nabla\cdot\vec{E}=-\nabla^2\phi=-4\pi e\int f d^3v. \tag{2}$$

The distribution function is assumed to depart only slightly from the zeroth order distribution, and the spatial dependence of the perturbation of the distribution is assumed to be given by the factor $\exp(i\vec{k}\cdot\vec{r})$. Equations (1) and (2) are linearized and then solved by taking the Laplace transform and following the procedure of Bernstein.[1] It is found that the Laplace transform of the potential is given by

$$\phi(s)=\left[-\frac{4\pi e}{k^2\omega_c}\int g(\vec{v},\vec{k},s)d^3v\right]/[1-Y(s)]. \tag{3}$$

In Eq. (3), s is the Laplace transform parameter, $\omega_c=eB/mc$ is the cyclotron frequency and $g(\vec{v},\vec{k},s)$ is a function related to the initial value of the perturbation of the distribution function. $Y(s)$ is given by

$$Y(s)=2\pi i\frac{\omega_p^2}{k^2}\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty}dv_z\int_0^{\infty}v_\perp dv_\perp \times\left\{\frac{\omega_c}{v_\perp}\frac{\partial F}{\partial v_\perp}\frac{nJ_n^2\left(k_\perp v_\perp/\omega_c\right)}{(s+ik_zv_z+in\omega_c)}+k_z\frac{\partial F}{\partial v_z}\frac{J_n^2\left(k_\perp v_\perp/\omega_c\right)}{(s+ik_zv_z+in\omega_c)}\right\}. \tag{4}$$

In Eq. (4), $\omega_p=(4\pi Ne^2/m)^{1/2}$ is the plasma frequency, k_z is the component of $\vec{k}$ along $\vec{B}$, and $k_\perp$ is the perpendicular component. Similarly, v_z is the component of $\vec{v}$ along $\vec{B}$ and $v_\perp$ is the perpendicular component. J_n is the Bessel function of order n. $F(v_z,v_\perp)$ is the zeroth order distribution. It is normalized so that its integral over all of velocity space is unity.

We are particularly interested in zeroth order distributions which cause the denominator of Eq. (3) to vanish for some values of s which have positive real parts. That is,

$$Y(s)=1 \quad \text{for } \mathrm{Re}(s)>0. \tag{5}$$

If Eq. (5) is satisfied then there will exist plas-

ma oscillations whose amplitudes increase exponentially. We shall exhibit two distributions for which this is true. First, we consider

$$F(v_z, v_\perp) = \frac{1}{2\pi}\,\delta(v_z)\,\frac{\delta(v_\perp - V)}{v_\perp}. \qquad (6)$$

Equation (4) becomes

$$Y(s) = -\left(\frac{\omega_p}{\omega_c}\right)^2 \sum_{n=-\infty}^{+\infty} \left\{ i\left(\frac{k_\perp}{k}\right)^2 \frac{n}{(s/\omega_c + in)}\left[\frac{1}{b}\,\frac{d}{db}\,J_n^2(b)\right] + \left(\frac{k_z}{k}\right)^2 \frac{1}{(s/\omega_c + in)^2}\,[J_n^2(b)] \right\}, \qquad (7)$$

where $b = k_\perp V/\omega_c$.

In order to determine whether unstable oscillations exist we use the Nyquist criterion.[2] That is, we note that $Y(s)$ defines a mapping of the right half of the s-plane onto a region of the $Y(s)$ plane, and the boundary of this region is the curve $Y = Y(i\omega)$, $-\infty < \omega < +\infty$. Equation (5) will be satisfied if the curve $Y = Y(i\omega)$ encloses the point $+1$. If we plot the Nyquist diagram of Eq. (7) we find that for a suitable choice of b, $k_\perp$, k_z, and (ω_p/ω_c) unstable oscillations exist. If $k_z = 0$, unstable oscillations exist only for $b > 1.84$ [the first maximum of $J_1(b)$].

The distribution given by Eq. (6) was first studied by Malmfors[3] who concluded that instabilities existed. However, an error in his work was found by Gross.[4] Malmfors and Gross considered only the case $k_z = 0$. Gross showed that there was no instability for $b \ll 1$ and conjectured that there was no instability for any b. Sen[5] was able to show numerically that there were instabilities for large b. Sen also considered only the case $k_z = 0$. Our analysis indicates that the greatest instability occurs when neither k_z nor $k_\perp$ are zero. Instabilities of this sort may, as Malmfors suggests, account for the noise in trochotrons reported by Alfven et al.[6]

We have also considered the distribution

$$F(v_z, v_\perp) = \frac{\alpha_z}{\pi^2 \alpha_\perp^2}\,\frac{\exp(-v_\perp^2/\alpha_\perp^2)}{v_z^2 + \alpha_z^2}. \qquad (8)$$

This distribution was chosen because it resembles the Maxwell-Boltzmann function and allows the integrals in Eq. (4) to be evaluated in terms of known functions. The isotropy of the distribution can be varied by changing $\alpha_\perp$ and α_z.

Substituting Eq. (3) into Eq. (4) gives

$$Y(s) = -\left(\frac{\omega_p}{\omega_c}\right)^2 \sum_{n=-\infty}^{+\infty} e^{-\lambda} I_n(\lambda) \times \left\{ \frac{i}{\lambda}\left(\frac{k_\perp}{k}\right)^2 \frac{n}{[(s + |k_z \alpha_z|)/\omega_c + in]} + \left(\frac{k_z}{k}\right)^2 \frac{1}{[(s + |k_z \alpha_z|)/\omega_c + in]^2} \right\}, \qquad (9)$$

where $\lambda = \alpha_\perp^2 k_\perp^2/2\omega_c^2$ and $I_n(\lambda) = I_{-n}(\lambda)$ is the modified Bessel function of the first kind. When $k_z = 0$ Eq. (9) may be shown to agree with a result of Bernstein.[7] There is neither instability nor damping. It may also be shown that there can be no instability for $k_\perp = 0$ although there will be Landau damping unless $\alpha_z = 0$. Instability can only occur when neither $k_\perp$ nor k_z is zero.

We have considered the intermediate case $k_\perp = k_z$ with $\lambda = 0.5$ and $\alpha_z = 0$. A plot of the Nyquist diagram showed that there were unstable oscillations for $\omega_p/\omega_c > 1.1$.

It is apparent from the structure of Eq. (9) that a nonzero value of α_z will decrease the instability or increase the damping of the oscillations.

The instabilities discussed here may have serious consequences for attempts to achieve controlled thermonuclear reactions. The instabilities are due to the anisotropy of the velocity distributions. In most thermonuclear devices anisotropies naturally arise. In high-energy injection devices such as the Oak Ridge DCX and the Russian OGRA the average particle velocities are greater perpendicular to the field than along the field. The same thing is true to a somewhat smaller extent in any machine that relies on magnetic mirrors for containment. Machines of the Stellarator type may produce anisotropies by magnetic pumping. A further study of these instabilities is in progress.

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†Operated by Union Carbide Corporation for the U. S. Atomic Energy Commission.

[1]I. B. Bernstein, Phys. Rev. 109, 10 (1958).

[2]James, Nichols, and Philips. Theory of Servomechanisms (McGraw-Hill Book Company, New York, 1947), p. 70.

[3]K. G. Malmfors, Arkiv Fysik 1, 569 (1950).
[4]E. P. Gross, Phys. Rev. 82, 232 (1951).
[5]H. K. Sen, Phys. Rev. 88, 816 (1952).
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[7]Equation (46) of reference 1.

DEVELOPMENT OF HYDROMAGNETIC SHOCKS FROM LARGE-AMPLITUDE ALFVEN WAVES

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Several authors, notably de Hoffmann and Teller,[1] have treated the conditions which obtain across a fully developed shock front in an ionized gas in the absence of external fields. Petschek[2] has derived the equations governing the growth in time of such a shock from a magnetosonic pulse of large amplitude. Ferraro[3] has discussed large-amplitude, circularly-polarized Alfvén waves and has found that such waves, if thermal motions are negligible and if there are initially no forces in the direction of propagation, will propagate undistorted in time. However, it is shown here that plane-polarized Alfvén waves of large amplitude develop rapidly into hydromagnetic shocks.

Consider the motion of an ionized gas consisting of electrons of mass m_- and positive ions of mass m_+, each of equilibrium density n_0 cm^{-3}, infinite in all directions, and immersed in a constant external magnetic field B_0 directed along the x-axis. Assume that (1) the gas remains electrically neutral to a high degree throughout the motion; (2) $n_0kT/B_0^2 \ll 1$, where T is the maximum temperature at any point in the gas; (3) the mean free path for collisions is $\gg$ all the characteristic lengths of the motion; (4) the displacement current is always $\ll$ the conduction current; and (5) $B_0^2/4\pi n_0(m_+ + m_-) \ll c^2$. The circumstances under which (1)-(5) apply are well known.[4]

Consider the following progressive pulse:

$$\vec{B} = B_0\hat{i} + B_y(x,t)\hat{j},$$
$$\vec{E} = E_z(x,t)\hat{k},$$
$$\vec{J} = J_z(x,t)\hat{k}. \qquad (1)$$

Note that the average electron and ion velocities in the x and y directions, V_x and V_y, are equal. The electric current is $\vec{J}$.

The subsequent motion will then be governed by the Boltzmann-Vlasov equations:

$$\frac{\partial f_-}{\partial t} + \vec{v}\cdot\nabla f_- - \frac{e}{m_-}(\vec{E} + \vec{v}\times\vec{B}) - \nabla_v f_- = 0, \qquad (2a)$$

$$\frac{\partial f_+}{\partial t} + \vec{v}\cdot\nabla f_+ + \frac{e}{m_+}(\vec{E} + \vec{v}\times\vec{B}) - \nabla_v f_+ = 0, \qquad (2b)$$

and by the Maxwell equations:

$$\nabla\cdot\vec{E} = 0, \qquad (3a)$$
$$\nabla\cdot\vec{B} = 0, \qquad (3b)$$
$$\nabla\times\vec{E} = -\partial\vec{B}/\partial t, \qquad (3c)$$
$$\nabla\times\vec{B} = 4\pi\vec{J}, \qquad (3d)$$

where

$$\vec{J} \equiv e\int f_+\vec{v}d^3v - e\int f_-\vec{v}d^3v, \qquad (4a)$$
$$\vec{V} \equiv [m_+\int f_+\vec{v}d^3v + m_-\int f_-\vec{v}d^3v]/(m_+ + m_-). \qquad (4b)$$

Note that (3a) and (3b) are automatically satisfied. Further, assume that $\partial f_\pm/\partial y = \partial f_\pm/\partial z = 0$.

Take zeroth-order moments of Eqs. (2) and add:

$$d\rho/dt + \rho\partial V_x/\partial x = 0, \qquad (5)$$

where ρ is the mass density, and where

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + V_x\frac{\partial}{\partial x}. \qquad (6)$$

Taking first-order moments gives, using assumption (2) and Eq. (3d),

$$\rho\frac{d\vec{V}}{dt} = \vec{J}\times\vec{B} = \frac{(\nabla\times\vec{B})\times\vec{B}}{4\pi}, \qquad (7)$$

the x-component of which is

$$\rho\frac{dV_x}{dt} + \frac{\partial}{\partial x}\left(\frac{B_0^2 + B_y^2(x,t)}{8\pi}\right) = 0. \qquad (8)$$

Taking the first moment of (2a) alone and passing to the limit $m_-/e \to 0$ yields[5]

$$E_z + V_xB_y - V_yB_0 = 0, \quad V_{z_-} = 0. \qquad (9)$$

Equations (3c), (5), (8), and (9) can be combined to give (at least through third order in B_y/B_0)

$$(B_0^2 + B_y^2)^{1/2}/\rho = \text{const.} \qquad (10)$$

It will now be apparent that Eqs. (5), (8), and (10) are nothing more than the equations for a nonlinear sound wave from ordinary gas dynamics, with the replacement of $(B_0^2 + B_y^2)/8\pi$ for the pressure, a frequent result in plasma dynam

ics. Equation (10) is the analog of the adiabatic relation $P/\rho^\gamma = \text{const}$, with $\gamma = 2$. The ordinary gas-dynamical theory can be applied; in particular, the distortion of the wave profile in a plot of $B_y^2(x,t)$ vs x can be followed until a vertical tangent forms (i.e., a shock front) by the usual geometrical construction. The local velocity of the wave motion is a,

$$a^2 \equiv \frac{d}{d\rho}\left(\frac{B_0^2 + B_y^2}{8\pi}\right) = \frac{B_0^2 + B_y^2}{4\pi\rho}, \qquad (11)$$

which, of course, approaches the Alfvén velocity as $B_y/B_0 \to 0$. Points of higher B_y^2 will gain on points of lower, and the pulse will steepen to a discontinuity. One point on a plot of B_y^2 as a function of x becomes vertical after a time t_s, which is readily shown to be[6]

$$t_s \approx \frac{2B_0(1 + B_y^2/B_0^2)^{3/4}(4\pi\rho_0)^{1/2}}{B_y\,|dB_y/dx|_{\max}} \qquad (12)$$

All variable quantities are evaluated at the point of maximum $|dB_y/dx|$ at $t = 0$. The shock time t_s is of the order of one period if B_y/B_0 is of the order of 1 and B_y is approximately sinusoidal at $t = 0$. Since the system of Eqs. (2)-(3) conserves entropy,[7] the discontinuity differs from acoustical shocks in that it is nondissipative; this is because the random or thermal effects have been neglected by assumption (2).

It is not clear what process ultimately limits the thickness of the shock; an eventual charge separation seems more likely than the dissipation of energy by viscosity of ordinary acoustical shocks.[2] It would also be of interest to calculate the stability of the motion.

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[3]V. C. A. Ferraro, Proc. Roy. Soc. (London) A233, 310 (1955).

[4]See, for instance, Bernstein, Frieman, Kruskal, and Kulsrud, Proc. Roy Soc. (London) A244, 17 (1958).

[5]Sometimes loosely called the "infinite conductivity" assumption, this is mathematically equivalent to making the electron Larmor radius small compared to the distances over which the fields vary appreciably. See Chew, Goldberger, and Low, Proc. Roy. Soc. (London) A236, 11 (1956).

[6]The time of steepening is worked out in detail for an acoustical wave by G. I. Taylor and J. W. Maccoll, in Aerodynamic Theory, edited by W. F. Durand (California Institute of Technology, Pasadena, 1943), Vol. 3, pp. 210-217. The calculation leading to (12) is similar.

[7]Ira. B. Bernstein, Phys. Rev. 109, 10 (1957).

EXCITATION OF OSCILLATIONS IN A PLASMA LAYER

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(Received December 15, 1958)

Recently the author developed a linearized theory of excited plasma waves,[1] in which electron plasma oscillations excited by an injected beam into a uniform plasma were discussed in detail. The dispersion relation in this case was obtained as follows:

$$(\omega_0/\omega_p)^2 = 1 + (3/2\beta^2) + (15/4\beta^4) + \ldots + \sigma\beta^2/(\xi - \beta)^2, \qquad (1)$$

where $\beta = \omega/ku_b$, $\xi = u_b/u_T$, $\omega_p^2 = n_p e^2/m\epsilon_0$, and $\sigma = n_b/n_p$. u_b and n_b denote uniform velocity and electron density in the beam respectively, while $u_T = (2\kappa T/m)^{1/2}$ and n_p are thermal velocity and electron density in the plasma, respectively. The relation (1) was derived without any ambiguity by using an approximation for $|\beta| \gg 1$. Although similar dispersion relations have been derived and discussed to some extent by several authors,[2-5] more exact and detailed characteristics of excited waves were needed for our purpose. Equation (1) was solved for an excited wave, where $\mathrm{Im}(\omega) = \gamma > 0$, the complex frequency being written as $\omega = \omega_0 + i\gamma$. Each wave component builds up from a ground level of initial disturbance proportionally to $\exp(\gamma t)\exp(kz - \omega_0 t)$. Except for very small values of σ, Eq. (1) must be solved numerically. The typical case with $\sigma = 0.1$ and $(u_b/u_T)^2 \to \infty$ is shown in Fig. 1, where the frequency ω_0 and the wave growth rate γ, both normalized by ω_p, and the ratio of plasma wave velocity $u_w = \omega_0/k$ to the beam velocity u_b are plotted against the wave number k. These characteristics are found to be not very sensitive to finite values of $(u_b/u_T)^2$.

The theory is applied to the excitation of a standing wave in a uniform plasma layer with a thickness D. The wavelength λ of the standing wave is determined by the relation $D = n(\lambda/2)$,

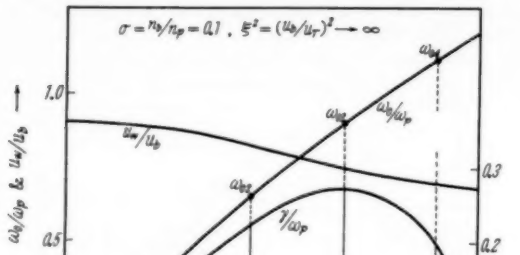

FIG. 1. Typical characteristics of excited wave in a plasma layer with a thickness D.

where n is an integer. Hence the wave number k has discrete values: $k = n\lambda/D$. Corresponding to a value of k, a wave with the frequency ω_{0n} is excited with the growth rate γ_n (Fig. 1). Since γ_n is proportional to the energy transfer from the beam to the plasma wave, it would represent the relative intensity for each excited wave within the validity of the linear approximation. When $\pi u_b/D\omega_p$ is not too small, it may be reasonable to assume that the wave actually observed corresponds to the mode with the maximum γ_n. For example, γ_3 is the maximum in Fig. 1 and therefore the wave with the mode $n = 3$ will be excited.

Now we consider the case when the parameter $\pi u_b/D\omega_p$ decreases continually. Then, in Fig. 1, γ_3 tends to be smaller while γ_4 becomes larger; the frequency ω_{03} is gradually lowered, exhibiting a frequency-pulling phenomenon. As soon as γ_4 exceeds γ_3, the mode number of the observed plasma wave changes abruptly from $n = 3$ to 4. At the same time, the frequency jumps from ω_{03} to ω_{04}. Further decrease of the parameter $\pi u_b/D\omega_p$ causes repeated jumps of the wave mode and frequency as represented in Fig. 2. Corresponding to the change of magnitude of γ_n, the intensity of the excited waves will become rather weak near the region where the mode jumps occur and will attain its maximum almost midway between jumps. Since every combination of changes of the quantities u_b, D, and ω_p brings about the variation of the parameter $\pi u_b/D\omega_p$, yielding the frequency-pulling and the

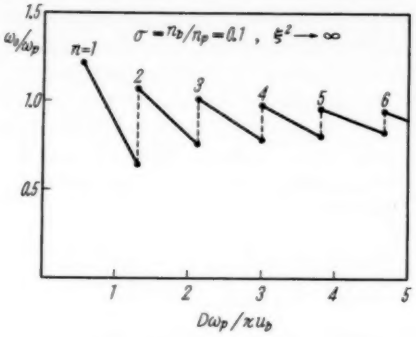

FIG. 2. The patterns of frequency and mode when the parameter $D\omega_p/\pi u_b$ varies.

mode frequency jump, the patterns represented in Fig. 2 are of great importance for the prediction of various phenomena based on our theory.

There are three simple cases in which the parameter $D\omega_p/\pi u_b$ varies. First is the case where ω_p and u_b remain constant and the thickness of the plasma layer D changes. Experiment corresponding to this have been performed by Looney and Brown[6] and by Bailey and Emeleus[7] (abbreviated hereafter as L-B and B-E, respectively). In the former, successive jumps of frequency and mode of standing waves were observed as the thickness of plasma sheath was varied. It corresponds to just what is shown in Fig. 2. In the latter (B-E), similar frequency-jump phenomena were displayed as a probe was moved away from the cathode. This can be well illustrated if we suppose that standing waves were excited in the region between the cathode and the probe. The intensity distribution in this case, too, is nicely checked by the theory. The second case is that in which ω_p is varied, while u_b, D, and σ are kept constant. It may be realized when the beam density is changed, the beam voltage remaining unchanged. Then D and σ may be practically regarded as constant. L-B have performed a measurement like this, which show jumps of frequency and mode of standing wave and variations of their intensities just as expected by the above theory. Looney and Brown themselves tried to illustrate these phenomena by means of a klystron-like mechanism. However, it seems more reasonable to explain the results by the standing-wave mechanism mentioned above. Third is the case in which u_b is altered, whereas ω_p and D are unaltered. This can also be expected to give rise to the same phenomena as the first case, although the cor-

responding experiment has not yet been carried out.

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ELECTRON SPIN RESONANCE OF ACCEPTOR STATES IN DIAMOND

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(Received December 15, 1958)

Previous work[1,2] reporting electron spin resonance in diamond has been concerned exclusively with paramagnetic centers produced by irradiation with fast neutrons. Using standard resonance techniques we have recently detected at room temperature a family of weak, narrow ($\Delta H \cong \frac{1}{3}$ oersted) resonance lines near $g=2$ in natural unirradiated diamonds. There are between 14 and 30 or more lines in the spectrum, depending on the orientation of the dc magnetic field with respect to the crystallographic axes; and these are contained in a total magnetic field spread of approximately 30 oersteds. The resonances are easily saturated, full resolution of the spectrum being obtained only when the input power to the resonant cavity ($Q_L \cong 5000$) is limited to a few microwatts. The diamonds, supplied by the Rough Diamond Company, New York City, displayed a yellowish coloration whose intensity varied from sample to sample.

It is proposed that the observed spectrum is due to bound aluminum acceptors. This interpretation is based on the following arguments: The results of previous spectrographic studies[3] of the naturally occurring impurities in diamond indicate that aluminum is one of the most frequently found elements in diamonds. It is furthermore likely[4] that aluminum with three outer electrons would enter the diamond lattice substitutionally and that a large fraction of the resulting number of "holes" would be bound to their acceptor atoms even at room temperature because of the large ionization energy of diamond as compared to germanium and silicon. One can account qualitatively for the observed spectrum by considering the spin resonance of an unpaired electron localized along one of the four tetrahedral bonds joining an aluminum atom with its nearest neighbor carbon atoms. A small amount of s-character at the aluminum nucleus of the wave function of the unpaired electron will give rise to six ($I_{Al} = \frac{5}{2}$) hyperfine lines ($\Delta M_I = 0$). The interaction of the quadrupole moment of the aluminum nucleus with the field gradient of the unpaired electron will add so-called "forbidden" resonances (eight transitions $\Delta M_I = \pm 1$, eight transitions $\Delta M_I = \pm 2$) to the spectrum. The lines for which $\Delta M_I = \pm 2$ are generally pronounced only when the angle θ between the dc field H_0 and the symmetry axis for the axially symmetric electric field (the Al-C bond direction) is approximately 90°. As there are four equally probable bond directions along which the "hole" can be oriented, there will be four dissimilar but overlapping sets of lines for an arbitrary orientation of the dc field with respect to the crystal axes. In such a case the spectrum is complex and difficult to analyze. If the crystal is oriented so that the dc field H_0 is parallel to a [100] direction of the crystal, thus making an equal angle ($\theta \cong 55°$) with each of the tetrahedral bonds, the four sets of lines should coincide. Experimentally this is the case (Fig. 1), and the 14 distinct lines observed in this orientation are probably the transitions $\Delta M_I = 0$, $\Delta M_I = \pm 1$.

Application of perturbation theory to reconstruct the spectrum theoretically is difficult in

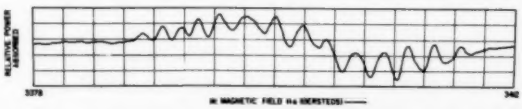

FIG. 1. Electron spin resonance spectrum of aluminum in diamond at 9478 Mc/sec (temp. = 295°K). The dc magnetic field is parallel to a [100] direction of the sample. (Note that for this orientation the line widths are greater than the $\frac{1}{3}$-oersted value stated in the text because of the superposition of several sets of lines).

this case because the hyperfine, quadrupole, and even nuclear magnetic energies are all of the same order of magnitude. The above model accounts for most of the features of the spectrum and is quite similar to the model proposed by Griffiths et al.[1] and by O'Brien and Pryce[2] to explain the resonances observed in irradiated (smoky) quartz where the effects were ultimately due to the presence of naturally occurring aluminum impurities.

The assumption in the above model that each hole is oriented along a definite tetrahedral bond direction may imply that in the diamond-Al system the spin precession frequency in a field of approximately 3 kilogauss is large compared to a molecular resonance frequency governing the rate at which the hole flips from one tetrahedral bond direction into another. A more detailed analysis of the observed resonances is in progress.

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[1]Griffiths, Owen, and Ward, in Defects in Crystalline Solids (The Physical Society, London, 1955).

[2]M. C. M. O'Brien, and M. H. L. Pryce, in Defects in Crystalline Solids (The Physical Society, London, 1955).

[3]F. A. Raal, Am. Mineralogist 42, 354-361 (1957).

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OSCILLATORY ETTINGHAUSEN-NERNST EFFECT*

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(Received December 23, 1958)

A number of metals which exhibit the de Haas-van Alphen effect show a similar oscillatory field dependence in their galvanomagnetic and thermomagnetic properties. Oscillations which are periodic in H^{-1} have been observed, in the case of one or more metals, in the magnetoresistance, thermal conductivity, Hall effect, and thermoelectric power.[1] Reported here are observations at liquid helium temperatures of such oscillations in the Ettinghausen-Nernst effect.

The sample was a single crystal of zinc in the form of a rectangular parallelepiped of dimensions 26.58×8.00×0.345 mm. With the crystal mounted vertically, the hexagonal axis lay in the horizontal plane at an angle of 44° with the normal to the large face. An electric current of one ampere passed between two pressure contacts at one end of the crystal, thus heating that end. With a heat current passing down the length of the crystal, the electric potential was measured between probes placed transverse to this current. This potential was amplified and fed to a recording potentiometer, as the magnetic field was increased at a uniform rate. A photograph of one recorder trace is seen in Fig. 1. The horizontal pips are not noise but signals superimposed on the recorder trace to indicate pres-

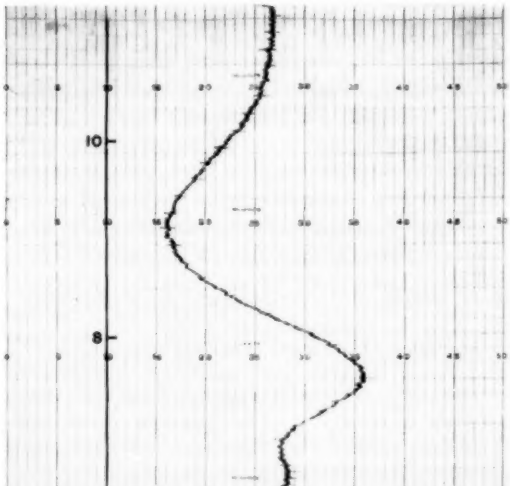

FIG. 1. A typical recording of the Ettinghausen-Nernst potential for uniformly increasing magnetic field.

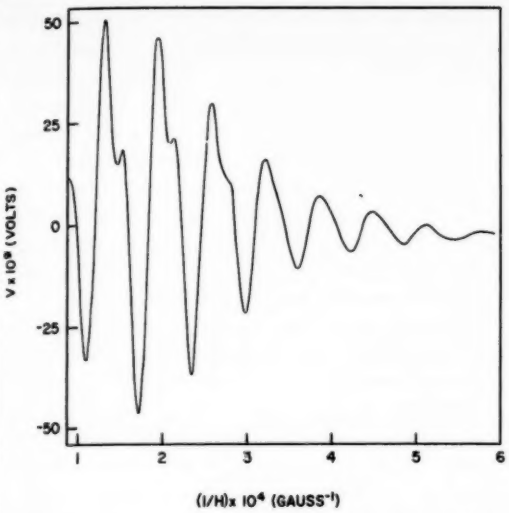

FIG. 2. The Ettinghausen-Nernst potential plotted against H^{-1}.

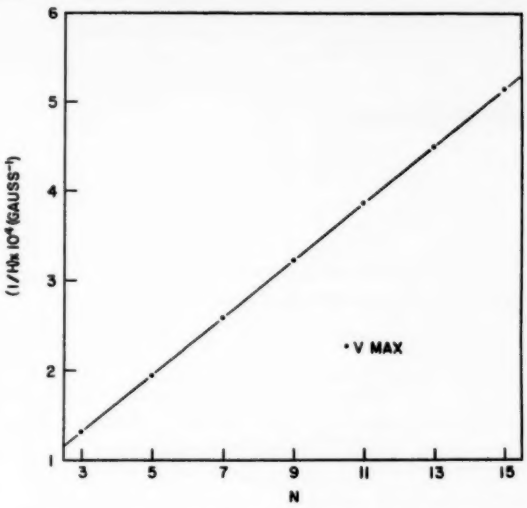

FIG. 3. Values of H^{-1} for maxima in Fig. 2 plotted against integers.

cribed values of the current in the magnet. Unlike the other oscillatory galvanomagnetic effect, the potential between the probes is seen to be purely oscillatory.

The data in Fig. 1 were taken with the magnetic field parallel to the hexagonal axis of the crystal and with the sample at 4.2°K. Similar data were taken at other temperatures between 2.19°K and 4.2°K. In Fig. 2 the Ettinghausen-Nernst potential is plotted against the reciprocal of the magnetic field yielding a curve which looks remarkably like similar plots for the susceptibility in zinc. The H^{-1} values of the maxima in this curve appear in Fig. 3 plotted against integers. The oscillations are seen to be periodic in H^{-1}. The period taken from the slope of this curve is 6.3×10^{-4} gauss^{-1} in agreement with the period of the Hall effect and magnetoresistance oscillations.[2]

As the sample was immersed directly in liquid helium, which also took part in the heat conduction, values of the Ettinghausen-Nernst coefficient could not be obtained. Also the effect was observable only at temperatures above the lambda point for liquid helium, where liquid helium is a rather poor heat conductor. It is estimated that data shown in Fig. 1 correspond to a heat flow of 8×10^{-3} watt at 3000 gauss and 4×10^{-3} watt at 8000 gauss. This decrease in heat flow is caused by a monotonic increase in the thermal magnetoresistance with rising field. Measurements of this effect are in progress on a crystal in a high vacuum and at liquid helium temperatures. The results of the more careful measurements will be reported later.

*Supported by the Office of Ordnance Research, U. S. Army.

[1]For a list of the metals in which each effect has been observed along with complete references, see J.-P. Jan, in Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, Inc. New York, 1957), Vol. 5.

[2]Ali, Grenier, and Reynolds, Bull. Am. Phys. Soc. Ser. II, 3, 304 (1958).

MEASUREMENT OF DIELECTRIC SURFACE CHARGE DENSITY AND TOTAL CHARGE DENSITY OF A CONDENSER

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(Received December 8, 1958)

The charge density at the surface of mica, placed between two condenser plates, in direct contact with the plates, and the total charge density of this condenser are measured as a function of the applied potential. A proposed interpretation of the experimental results is based on the energy band theory of solids.

The experimental arrangement is shown in

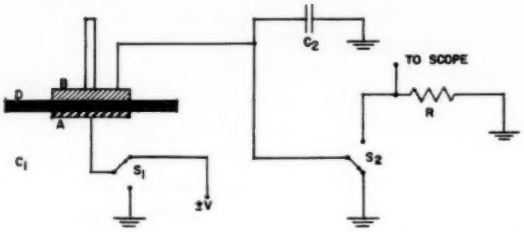

FIG. 1. Experimental arrangement. The mica under investigation is placed in the condenser C_1.

Fig. 1. The mica D under investigation is placed in the condenser C_1. The fixed condenser plate A is made of silver paint. The portable condenser plate B is made of a 10 ohm cm n-type single-crystal silicon. The mating surface of mica is freshly peeled, and that of the silicon is optically polished and freshly etched in CP-4 just before the experiment.

The total charge density of the condenser is measured as follows: A potential V with respect to ground is applied to A. B is discharged through a constant resistance R to ground. The oscilloscope is used to record the current as a function of time. The total charge density Σ_t at the B side of the condenser is obtained by measuring the area under the curve in which current is shown as a function of time. The result of Σ_t versus V is indicated by the solid line in Fig. 2.

The measurement of the charge density at the mica surface is based upon the following considerations: Suppose a condenser is charged in vacuum with field intensity $\vec{E}$; then the metal plates have a surface charge density $\Sigma_0 = \pm E/4\pi$. When the dielectric is inserted, polarization charge densities $\pm P$ are induced on the surfaces of the dielectric. An additional charge density Σ_p flows from the circuit to the condenser plates to neutralize the polarization charge. If $\vec{E}$ is small, P is proportional to E. Thus the total charge density on the metal plate is $\Sigma_t = \Sigma_0 + \Sigma_p = \pm(E/4\pi + P) = \pm\epsilon E/4\pi$. If one metal plate is replaced by a semiconductor plate, under the same field intensity, the semiconductor plate has the same surface charge density. If this charge density is provided by changes in the number of mobile holes or electrons, there will be a change in conductivity accompanying a change in E. This is called the field effect.[1] In the field-effect experiment,[2,3] the charge density Σ_{sc}, in the form of mobile holes and electrons, is determined by measuring the changes of conductivity of the semiconductor by a given $\vec{E}$. But Σ_{sc} thus obtained is much less than Σ_t. This discrepancy is explained by postulating the existence of surface states at the surface of the semiconductor. A large portion of the induced charge Σ_t appears in these states and does not contribute to the conductivity. The exact amount is $\Sigma_s = \Sigma_t - \Sigma_{sc}$. In the cases described above, the total charge density is considered to be entirely on the condenser plates and no charge transportation between condenser plate and the dielectric is assumed. The positive and negative charges in the dielectric, under the influence of an electric field, are merely displaced, but do not separate from each other. If this is true, the following phenomenon should be observed. If after the condenser is charged, B is moved far away from A, and A is grounded, and B is then insulated from ground and replaced in its original position, no charge should remain

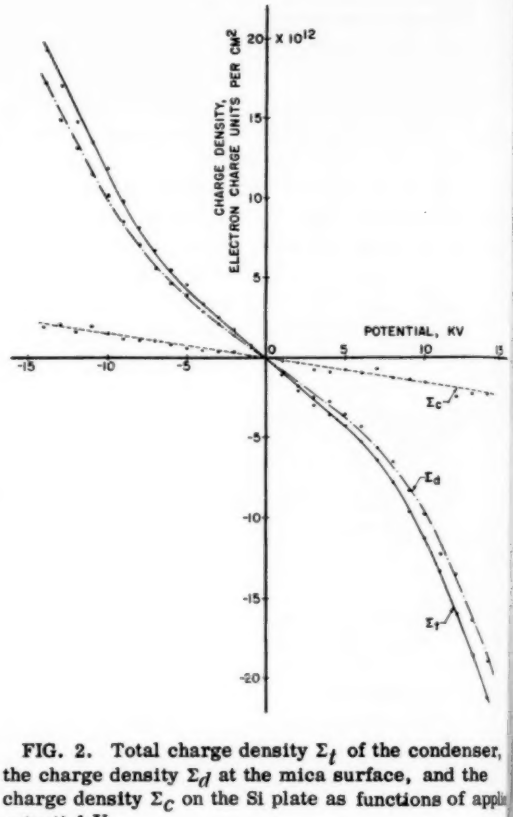

FIG. 2. Total charge density Σ_t of the condenser, the charge density Σ_d at the mica surface, and the charge density Σ_C on the Si plate as functions of appli potential V.

in the condenser. Since the dielectric is not ferroelectric, does not have permanent electric moment, and both plates are grounded before being again put together, the field intensity as well as the polarization should be zero. But the measurement shows that a considerable portion of Σ_t remains in the conductor.[4] This is possible only if, during the charging of the condenser, this portion of Σ_t is transferred from the silicon plate to the mica surface. During the removal of B, the charge at the mica surface, bound to the field produced by A, still remains on the mica surface. Therefore, Σ_d, the charge density on the mica surface, is obtained by measuring the charge still stored in the condenser. The result of Σ_d versus V is indicated by the broken line in Fig. 2.

The net charge density $\Sigma_c = \Sigma_t - \Sigma_d$ on the silicon plate is represented by the dotted line. The measurement obtained by using copper and 30 ohm cm n-type single-crystal Ge, respectively, as one of the condenser plates also has been made; the result is essentially the same as using Si.

If we assume that the positive ion cores of the lattice points of the semiconductor and the dielectric are fixed, the charge transferred from the semiconductor to the surface of the dielectric is in the form of electrons added or removed from the dielectric surface, depending upon the sign of the potential applied. According to the band theory, the valence band of the dielectric is completely filled and the conduction band is completely empty. If the added electrons had to go into the conduction band, or the abstracted electrons removed from the dielectric had to come from the valence band, conductivity in the dielectric would have resulted. Such is contrary to experience. To explain this fact, we may assume[5] that, at the dielectric surface, some localized electronic states exist in the forbidden energy gap of the dielectric. These states are partially filled, we can empty or fill them without affecting the insulating properties of the dielectric. The occupation of these states with respect to the applied potential can be fitted within the experimental error by the following empirical form $\Sigma_d = -A \sinh \alpha V$, where A and α are constants.

The author wishes to express his gratitude to Dr. Paul L. Copeland, who has given generously of his counsel and encouragement during the investigation described above.

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[4]R. Tomaschek, Grimsehls Lehrbuch der Physik (B. G. Teubner, Leipzig and Berlin, 1932).
[5]I. Tamm, Physik. Z. Sowjetunion 1, 733 (1932).

MULTIPLE TRAPPING SITES FOR HYDROGEN ATOMS IN SOLID ARGON*

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The electron spin resonance (ESR) spectrum of hydrogen atoms, produced by an electric discharge in H_2 and trapped in an argon matrix at 4.2°K, was shown to be a simple doublet.[1] However, an ESR spectrum of three doublets has recently been observed as a result of photolysis of 1% hydrogen iodide in argon at 4.2°K by resonance radiation from a low-pressure mercury lamp. The low-field components of the three doublets are shown in Fig. 1. One of the doublets, whose low-field component is labelled A in the figure, has a field separation of 507.4 oersteds. This is precisely the same as found in the earlier deposition experiment. The other two doublets (corresponding to B and C) have field separations of 508.4 and 515.8 oersteds, respectively.

The fact that the six lines in the spectrum are identified as three independent doublets follows

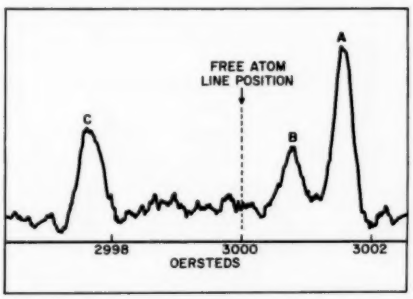

FIG. 1. Low-field components of ESR spectrum of hydrogen atoms from photolysis of hydrogen iodide in argon at 4.2°K. Microwave frequency = 9177.5 Mc/sec.

directly from the observation that each doublet shows a different temperature dependence. When the sample was warmed up very slowly from the initial temperature of 4.2°K, the components of doublet *C* gradually decreased and became unobservable at about 12°K, whereas the remaining lines substantially maintained their peak intensities while becoming somewhat narrower. In a similar manner, the components of doublet *B* disappeared at about 23°K and those of doublet *A* at about 39°K.

Since none of the above-mentioned doublets can be attributed to iodine atoms, which, in their free state, are expected to give an entirely different spectrum in another field region, we treat each of the observed doublets as the ESR spectrum of hydrogen atoms in a specific type of environment. From the field positions for each doublet and the microwave frequency, the effective g-factors and hyperfine coupling constants have been calculated by applying the Breit-Rabi formula. The results for all three doublets are tabulated in Table I, together with the known constants for the free hydrogen atom. Doublets *A* and *B* have approximately the same g-factor as the free atom, while the g-factor for doublet *C* is appreciably smaller. It is particularly noteworthy that doublets *A* and *B* have hyperfine coupling constants smaller than that of the free atom while doublet *C* has a larger hyperfine coupling constant.

It seems reasonable to interpret the three doublets as the ESR spectra of hydrogen atoms in three different lattice sites in an argon matrix. The stable crystalline form of argon at 4.2°K is face-centered cubic. Recent x-ray diffraction data[2] show that under the conditions of our experiment, argon is deposited as a polycrystalline material. In a face-centered cubic lattice there are three possible sites for a hydrogen atom: the substitutional, the octahedral, and the tetrahedral. In addition, there are many crystalline defects where hydrogen atoms could be trapped. There are thus a number of nonequivalent sites where hydrogen atoms could be placed. At the present time we are unable to assign the doublets to specific lattice sites.

Since doublet *A* has been observed in both the deposition and the photolysis experiments, it may be assumed that the occupation of site *A* can be achieved by a hydrogen atom with an energy corresponding to room temperature (~0.03 ev) or less. It should be noted for completeness that the line positions of doublet *B* fall very close to those of hydrogen atoms in an H_2 matrix.[1] It is believed that this is an accidental coincidence since it is extremely unlikely that H_2 was present in the experiment. Occupation of the sites giving rise to doublets *B* and *C* is probably made possible by the high energy content (upper limit ~3.6 ev) of the released atom in the photolytic experiment.

One major difference among the sites is revealed by their different effects on the hyperfine coupling constants of trapped hydrogen atoms. By the relation between the hyperfine coupling constant and electron density at the nucleus, one class of sites (*A* and *B*) gives a decreased electron density (negative shift) and the other class (*C*) gives an increased electron density (positive shift) as compared with the free state value. These two classes of sites must cause very different perturbations to the electronic wave function and, perhaps not too surprisingly, show

Table I. g-factors, hyperfine energy separations (ΔW), and upper temperature limits (T_l) for hydrogen atoms at various sites in solid argon.[a]

Site	g_J	ΔW (Mc/sec)	Deviation of ΔW from free value	T_l
free state[b, c]	2.002256(24)	1420.40573(5)	...	...
A	2.00220(8)	1413.82(40)	-0.46%	39°K
B	2.00224(12)	1416.31(80)	-0.29%	23°K
C	2.00161(8)	1436.24(40)	+1.15%	12°K

[a]The numbers in parentheses indicate experimental and/or conversion errors in the last figures of the associated values.

[b]P. Kusch, Phys. Rev. 100, 1188 (1955).

[c]R. Beringer and M. A. Heald, Phys. Rev. 95, 1474 (1954).

different degrees of thermal stability (see Table I). That a similar phenomenon can happen to S-state atoms in a different manner is brought out by recent reports[3] on the positive and negative frequency shifts produced by various buffer gases on the hyperfine coupling constant of Cs. Photolytic experiments with hydrogen atoms in other inert matrices are in progress.

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*This work was supported by the Bureau of Ordnance, Department of the Navy.

[1]Jen, Foner, Cochran, and Bowers, Phys. Rev. 112, 1169 (1958).

[2]H. S. Peiser, National Bureau of Standards (private communication).

[3]M. Arditi and T. R. Carver, Phys. Rev. 112, 449 (1958); Beaty, Bender, and Chi, Phys. Rev. 112, 450 (1958).

MEASUREMENT OF FAST LUMINESCENCE DECAY TIMES*

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(Received December 16, 1958)

Measurement of luminescence-decay curves of organic scintillators has important applications both for radiation chemistry and for nuclear physics. The basic problems of such work are amplification of a light signal and time measurement. The usefulness of the measurements depends on the time resolution and the number of half-decay intervals over which a decay curve may be measured. In all previous work[1-4] in this field, essentially the first stage of the measurement apparatus has involved amplification via a photomultiplier. Such technique limits subsequent time resolution to the transit-time spread of the signal in a photomultiplier[5] (i.e., at least 1 mμsec). In experiments in which the time measurement is effected by display of the scintillation signal on a cathode-ray tube, the signal is always markedly deteriorated after a decay of about one decade because of processes inside the photomultiplier and the connecting cable.[6,7] On the other hand, the so-called phase-shift method involves an assumption as to the form of the decay curve.

In the method here reported, which represents a new approach to the problem, the time "point" of the light signal is first established and the signal is thereafter amplified. In this way both time resolution and measurements at the far tail of the decay-curve are improved.

Figure 1 shows the apparatus used: the pulse generator, *PG* (Spencer-Kennedy Laboratory Model 203) delivers fast-rising positive square pulses into two 95-ohm coaxial lines. One of these pulses opens the grid of a 30-kv x-ray tube.[2] X-rays penetrate through a thin target into the scintillators. The scintillation light passes through an aperture A_1 (4 mm × 12 mm) and strikes the photocathode of an image converter, *IC* (RCA developmental type C73435B).[8] The photoelectrons there produced are accelerated and focused onto the luminescent screen. If the resultant light signal on the screen is directly in front of aperture A_2 (3×10 mm), the photomultiplier, *PM*, is energized. Normally, a dc bias voltage on the deflection plates of the image converter so controls the electron beam

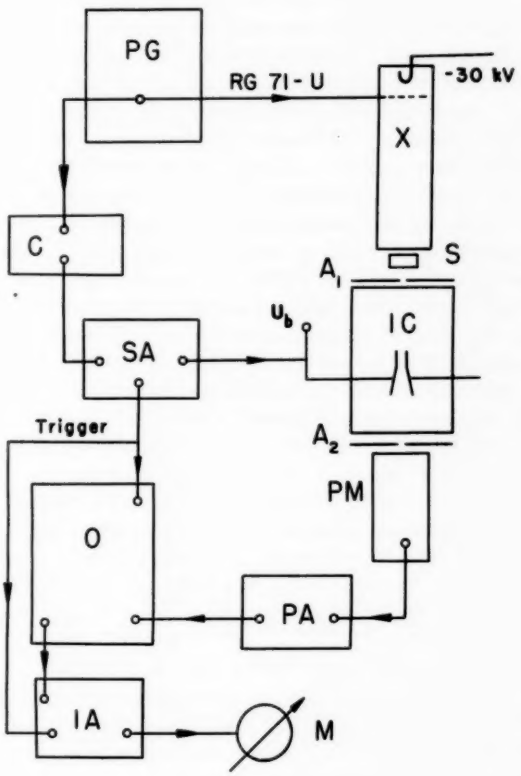

FIG. 1. Block diagram of apparatus for measurement of decay curves.

that no signal reaches the photomultiplier. A second pulse out of the pulse generator operates the sweep amplifier, *SA*, which produces a sweep speed of about 100 v/mμsec on the deflection plates of the image converter. Only those electrons which pass between the deflection plates at a time when the sweep voltage is about 0, and therefore hit the screen of the image converter directly in front of aperture A_2, will register their light signal on the photomultiplier.

Sweep speed together with the width of the apertures and deflection sensitivity give a calculated time resolution in the present apparatus of 0.5×10^{-9} sec. The signal to the sweep amplifier may be delayed by any desired time relative to the signal to the x-ray tube by a set of cables, *C*, of different length. It is thus possible to measure the amount of light released from the scintillator in a time interval Δt ($=0.5\times10^{-9}$ sec in our case) at a certain variable time after the x-ray pulse starts. Subsequent detection of this signal by the photomultiplier, *PM*, preamplifier, *PA*, and oscilloscope, *O*, is then no longer complicated by need for ultrafast response.

The shape of the signal is, of course, representative of long decay of the image converter fluorescent screen. Weak fluorescent signals exhibit a statistical fluctuation. Consequently, precise readings were taken from a phase-sensitive, integrating amplifier, *IA*, and a dc meter, *M*. For elimination of noise, the amplifier, triggered by the sweep amplifier, *SA*, is adjusted to respond only within 50 μsec of the trigger; for reduction of statistical fluctuation, the signals are integrated over a period of 4 sec (i.e., about 240 signals in this work).

Figure 2 shows, on a semilogarithmic plot, the intensity-time relationship for light emitted by different scintillators under excitation by an x-ray signal of about 10 mμsec length. The liquid scintillator shows a simple exponential decay over two decades, the commercial plastic scintillators show in each instance two different decay times, the second of which reflects the presence of some longer lived scintillator component.

Consideration of the time resolution of the apparatus indicates that, in its present state, the rise and cutoff time of the x-ray signal (about 1.2 mμsec) is the only limiting factor. Both emission of x-rays and emission of photoelectrons are very fast processes[9;10] ($<10^{-10}$ sec after excitation). Also the transit-time spread

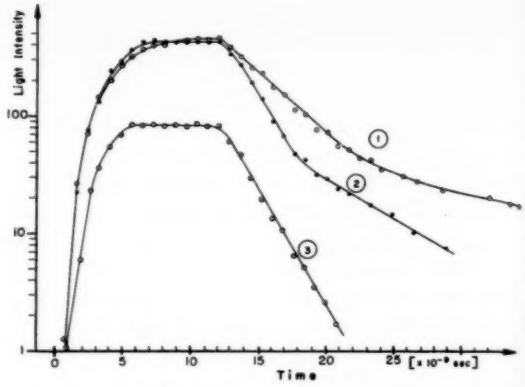

FIG. 2. Decay-curves of scintillators. (1) Plastic scintillator "Sintilon," National Radiac; τ initial $=4.2$ mμsec. (2) Plastic scintillator; $\tau_1=2.3$ mμsec, $\tau_2=6.3$ mμsec. (3) Liquid scintillator, 5 g/liter *p*-terphenyl in benzene; $\tau=2.2$ mμsec.

inside the image converter, *IC*, is rather small (estimated $<2\times10^{-10}$ sec) because of high acceleration voltage (5-15 kv) and a small effective photocathode. Consequently, the time resolution of measurement of decay-times by this method is well below 1 mμsec. Presently, there is no scintillator known to have a decay time short enough to verify this conclusion experimentally.

We are greatly indebted to Mr. R. K. Swank and the Argonne National Laboratory for the loan of the x-ray tube, which greatly facilitated the progress of this work.

*Contribution from the Radiation Project operated by the University of Notre Dame and supported in part by a U. S. Atomic Energy Commission Contract.

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[5]M. H. Greenblatt, IRE Trans. on Nuclear Sci. NS-5, 13 (1958).

[6]R. V. Smith, IRE Trans. on Nuclear Sci. NS-3, 120 (1956).

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X-RAY MEASUREMENT OF THE DISTRIBUTION OF ELECTRONS IN IRON AND COPPER

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(Received December 15, 1958)

In a recent publication Weiss and DeMarco[1] have reported an x-ray measurement of the number of $3d$ electrons in the metals Cr, Fe, Co, Ni, and Cu. Their results indicate that the metals with body-centered cubic structures have outer electron distributions which depart considerably from atomic $3d$ states. This letter communicates briefly the results of some present measurements of the atomic scattering factors of iron and copper which are not in agreement with Weiss and DeMarco.

The atomic scattering factors were obtained from the integrated intensities of Bragg reflections of copper and iron. One of the most important considerations in the x-ray measurement of scattering factors is the correction for extinction. There is at present no reliable theoretical method to correct for extinction. Hence, to make an accurate measure of scattering factors, one should perform the experiment in such a manner as to render extinction effects negligible. There is good evidence[2,3] that in a cold-worked metal, extinction effects can be made negligible, and therefore the atomic scattering factors were measured with cold-worked powders of iron and copper.

X-ray intensities were measured with monochromatic Fe K_α radiation. Carbonyl iron and electrolytic copper powders with a particle size 3-5μ in diameter were pressed into briquets in a highly polished mold with pressures ranging from zero to 60 000 lb/in.^{2}. At the higher pressures, the powders deformed plastically, and the copper briquets had a mirror surface with about a 3μ variation in flatness, estimated microscopically. The integrated intensities of corresponding reflections from all specimens agreed to within 6%. This includes specimens held together only by a bonding agent and those pressed with 60 000 lb/in.^{2}. As the molding pressure was increased, the half-width of the reflections increased, and the α_1-α_2 doublet resolution decreased. This indicates that the degree of cold work increased with increased molding pressure. Loose powders cold-worked in a small oscillating ball mill before being pressed into briquets gave intensities for all reflections within the 6% spread. Although there was considerable variation in the surface roughness of the different specimens, the variation in intensity was within experimental error.

The intensities of the five copper reflections relative to one another agreed to within 5% of the expected theoretical ratios. Similarly, the four measured iron reflections were also found to be internally consistent. This agreement shows that extinction and surface roughness are negligible for these powders under these experimental conditions. To put the iron and copper intensities on an absolute basis, the intensities from a finely pulverized rock salt specimen were measured under the same conditions as the metal powders. Using the tabulated atomic scattering factors for Na^+ [4] and Cl^-,[5] a parameter which effectively measures the primary beam intensity can be calculated. Knowing the primary beam intensity, an experimental value for the atomic scattering factors for copper and iron can be calculated from the observed intensities. These results are given in Table I,

Table I. Measured values of the atomic scattering factors for iron and copper standardized against sodium chloride.

NaCl			Copper			Iron		
hkl	Fitted	Theory[a,b]	*hkl*	Observed	Theory[b]	*hkl*	Observed	Theory[c]
200	23.0	21.47	111	22.5	22.16	110	19.3	18.77
220	18.4	18.21	200	20.6	20.78	200	15.8	15.57
222	15.9	16.17	220	17.0	16.78	211	14.0	13.45
400	15.2	14.76	311	15.1	14.76	220	12.3	11.91
420	13.2	13.70	222	14.9	14.20			
422	12.8	12.85						

[a]See reference 4. [b]See reference 5. [c]See reference 6.

together with the theoretical f_{Cu}[5] and f_{Fe}.[6] The experimental f values for NaCl obtained from the intensity parameter are also included. The (110) reflection of iron and the (111) of copper were carefully measured for three different pairs of specimens. Assuming $f_{(111)}$ of Cu[4,5] to be 22.16 (the value calculated from free-atom wave functions), the measured values of $f_{(110)}$ of iron are in the range 18.9 ± 0.1.

The dispersion corrections for Fe K_α radiation were calculated from Parratt and Hempstead[7] and are $\Delta f' = -2.45$ for iron and $\Delta f' = -1.51$ for copper. (The imaginary part, $\Delta f''$, has a negligible contribution.) To provide a cross check on the dispersion corrections, the iron (110) and copper (111) reflections were measured with unfiltered molybdenum radiation for which the correction is small and of opposite sign to that for iron radiation. Two pairs of specimens gave 18.8 and 18.9 for $f_{(110)}$ of iron relative to copper. These are in good agreement with the Fe K_α results.

Considering the consistent internal agreement with theory of all the measured reflections of either iron or copper and the external agreement with the rock salt intensities, it is felt that the validity of the assumptions of negligible extinction and surface roughness absorption has been demonstrated.

Using the calculated free-atom scattering factors for copper and sodium chloride, the value of f for iron at $\sin\theta/\lambda = 0.247$ [the (110) reflection] is estimated at 18.9 ± 0.2 relative to copper and 19.3 ± 0.6 relative to NaCl, while the free-atom calculation[6] gives 18.77. Thus, one may conclude that within experimental error the observed scattering factor for iron agrees with that calculated from the wave functions of the free atom.

If one chooses to ascribe any difference between observed and theoretical f to an excess or deficiency of $3d$ electrons, this may be done as follows: The contribution of each iron $3d$ electron to f at $\sin\theta/\lambda = 0.247$ is very nearly 0.60 unit.[6] Hence, in terms of these electrons, iron has 6.2 ± 0.4 $3d$ electrons relative to copper and 6.9 ± 1.0 relative to NaCl. Similarly, the contribution of each copper $3d$ electron to $\sin\theta/\lambda = 0.240$ [the (111) reflection] is approximately 0.62.[1] The measured value of $f_{(111)}$ of copper relative to NaCl is 22.5 ± 0.6 (free-atom value = 22.16), which, in terms of $3d$ electrons, is 10.5 ± 1.0.

Taking into account the estimated experimental errors, it is concluded that, to an accuracy of about one $3d$ electron, the number of electrons in metallic iron and copper is not different from that in the free atom. This contrasts with the value of (2.3 ± 0.3) $3d$ electrons for iron reported by Weiss and DeMarco from single-crystal measurements.

A more detailed report will be submitted shortly to the Physical Review.

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ELECTRON EMISSION FROM BREAKDOWN REGIONS IN SiC p-n JUNCTIONS

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We have measured electron emission currents from reverse-biased p-n junctions in SiC. In 20 samples the maximum emission ranged from 10^{-12} amp to 10^{-6} amp. In each case the emission depended strongly on sample preparation. Best results were obtained by heating the sample at 270°C in vacuum for several hours.

Burton[1] has reported similar emission from a reverse-biased junction in Si, but only after the addition of cesium to the sample tube. Evidently the cesium depresses the Si work function enough to permit the escape of some of the energetic electrons known to be present during breakdown. The high energy gap of SiC ($E_G = 2.86$ ev)[2] ensures the presence of correspondingly high-energy electrons at breakdown (for impact ionization electron energies of about one and a half times the gap are necessary).[3] We therefore expect to have conduction band electrons with energies in the neighborhood of 4.3 ev, an amou

greater than the estimated electron affinity[4] of 4 ev. Hence electron emission should occur from a clean surface if the breakdown region is sufficiently close to it.

The samples used were similar to those on which other measurements have been made.[5] The breakdown radiation, originating in small blue spots (~1 μ diam) is briefly described in reference 5. The spectrum extends to photon energies of about 4.6 ev. In most reversed-biased junctions many blue spots are observed, both within the junction area and at its periphery. It is thought that electrons may be emitted primarily from those spots which are very near the crystal surface. The sample was placed in a holder in such a way that a tungsten wire contact could be placed on the highly conducting n-type surface. It is at this surface that the blue spots appear. The sample holder and crystal were then almost surrounded by an Inconel can to which the positive collecting voltage was applied. One of the crystal contacts was a common terminal of both the crystal current and electron emission circuits.

A vibrating-reed electrometer was used to measure the electron emission current, which is shown in Fig. 1 as a function of the reverse current through sample $E4$. This sample had been baked at 270°C for 24 hr in a high vacuum, and then sealed off. After the first application of reverse voltage to the junction, the electron emission current was observed to increase many orders of magnitude during an incubation period of several minutes. Subsequently, the emission response to reverse bias was instantaneous, and reproducible within a factor of two, even after a waiting period of days. Such incubation periods are observed for all samples, and are especially long in those which have not been baked in air or vacuum. A surface change caused by the electron emission itself may be the reason for the great increase of emission during the incubation period.

Because breakdown occurs at small spots, the reverse characteristic of Sample $E4$ is too "soft" to define a breakdown voltage. Over the range of currents shown in Fig. 1, the reverse voltage increases from 7 to 38 volts. Breakdown radiation first appears to the dark-adapted eye, at a single peripheral spot, when the junction current is about the same as that at which electron emission begins. Other spots appear as the current is increased, until about 30 are visible at the highest reverse currents. Perhaps only a few of these contribute an appreciable number of electrons. Therefore the reverse current may not be a very significant variable against which to plot the emission. The total area of all spots is estimated to be about 10^{-6} cm^2, and hence the maximum emission current density is probably more than 1 amp/cm^2.

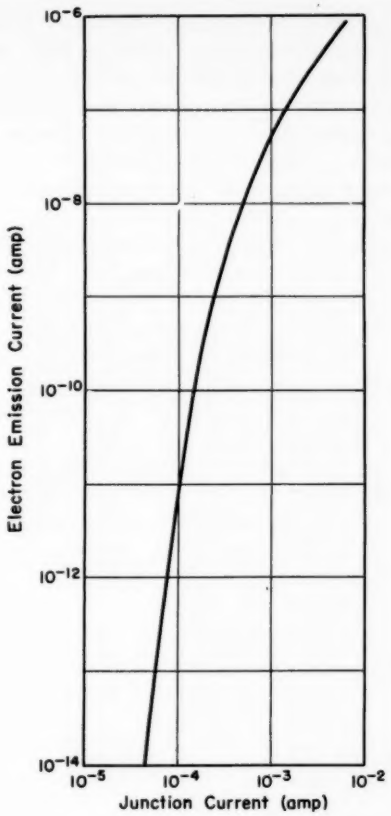

FIG. 1. Electron emission current vs reverse junction current for sample $E4$, with 135 volts on the collecting electrode.

As a function of collecting voltage, the electron emission at 1 ma reverse current obeys Child's law up to 100 volts, if the voltage zero is chosen as that at which the emission current goes to zero. This is a voltage intermediate between those at the crystal contacts. Above 100 volts, the emission current begins to saturate.

The electron current is not caused indirectly by the breakdown radiation through a photoemission mechanism, as the following observations show. (a) Several samples emitted many more electrons than photons. (b) An oxide film on the

surface may stop electron emission, but does not noticeably affect the light emission. (c) An ultraviolet lamp shining through the Pyrex envelope onto the Inconel electrode assembly did not excite any photocurrent ($<10^{-14}$ amp). Not more than 1% of the breakdown photons have energies greater than those received from the lamp.

The possibility of thermionic emission is excluded by the observation that there is good agreement between the measurements presented here and others in which the crystal current was pulsed. The electron emission response could be observed for a single 10-μsec pulse of crystal current. Circuit parameters prevented observation of still shorter pulses.

The electron emission of a given sample was found to depend strongly on sample treatment. For example, emission could be suppressed by rinsing the sample in water, then restored by heating the sample to 300°C in air or vacuum. Such effects are not unexpected; surface films and their effects on the work function are obviously important. Heating in air at 800°C for 1 hr also suppressed electron emission, probably because of the formation of an oxide layer[6] of less than 100 A. The emission was again restored by removing the oxide film with HF, followed by heating in air to 300°C. A later report will give more details on surface treatments, including cesium addition.

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NUCLEAR RESONANCE FLUORESCENCE IN Mg^{24}†

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A gamma ray emitted from a nucleus initially at rest has an energy which is less than the energy difference between the levels by an amount equal to the recoil energy of the emitting nucleus. In addition, if the gamma ray is to be resonantly scattered by a nucleus of the same kind, a like amount of recoil energy must be given to the scattering nucleus. Thus the nuclear resonance scattering cross section is extremely small unless this energy deficit is restored.

The 15-hour Na^{24} decays to Mg^{24} by beta emission with an endpoint of 1.39 Mev followed by a gamma cascade with energies of 2.76 Mev and 1.38 Mev. Pollard and Alburger[1] attempted to restore the resonance condition for scattering of the 1.38-Mev gamma ray by making use of the recoil from the previous radiations. No resonant scattering was observed and it was concluded that the 1.38-Mev level had a width of less than 0.01 ev. Recently Burgov and Terekhov[2] successfully used a coincidence method to observe the resonant scattering of those 1.38-Mev gamma rays which have the resonance condition restored by recoil due to the preceding 2.76-Mev gamma ray. The resonance effect was small and a direct cross-section measurement was not possible. However a lower limit (1.6×10^{-4} ev) was placed on the width of the 1.38-Mev level.

A large resonance effect has now been observed and preliminary measurements of the level width have been made. If the emitting nucleus is assumed to be at rest before emission of the gamma cascade, it is not difficult to show that the resonance condition for the second gamma ray can be restored by the recoil of the preceding gamma ray when the angle between the emission direction is given by $\cos\theta = -E_{\gamma 2}/E_{\gamma 1}$. In the particular case of Mg^{24}, $E_{\gamma 1} = 2.76$ Mev, $E_{\gamma 2} = 1.38$ Mev, and it is possible to observe resonant scattering of the 1.38-Mev gamma ray when the angle between it and the 2.76-Mev gamma ray is about 120°.

Pure sodium hydroxide was irradiated for 4-hour periods in the Ford Nuclear Reactor of the University of Michigan. The sources consisted of dilute (1.3 normal) aqueous solutions of NaOH sealed in Lucite containers.

A conventional fast-slow coincidence circuit with an effective resolving time of 15 millimicroseconds was employed in the present measurements. The 2.76-Mev gamma ray was detected by a 5 in. diameter × 4 in. long NaI(Tl) crystal mounted on a DuMont 6364 phototube. A lead collimator was used to restrict the angular width of the gamma ray beam entering this crystal and a pulse-height analyzer was set integrally to accept only the 2.76-Mev gamma ray.

A 1 in. high by 7/16 in. wide rectangular hole was cut between the parallel flat faces of a 2 in. diameter × 2 in. long cylindrical NaI(Tl) crystal. A bar of magnesium metal was placed within this well and the crystal was mounted on an RCA 6342 phototube. The resultant scintillation counter had a pulse height resolution (for direct radiation) of 10% at 1.38 Mev. A lead collimator limited the gamma rays entering this crystal to those which are scattered from the magnesium bar in the center. A differential analyzer was set to accept a narrow range of pulses above the center of the 1.38-Mev photopeak. Thus pulses due to Compton scattering from the magnesium were largely eliminated.

The coincidence rate was observed as a function of angle over a small range of angles centered at 120°. A sharp peak was observed, which, within the limits of experimental error, had an angular width as narrow as that of the collimating system (2.6° as measured by annihilation radiation). The coincidence rate at the peak was consistently two or three times the rate measured at neighboring angles off the peak (depending mainly on the sharpness of the pulse height discrimination).

Since the observed resonance effect was so large, it was possible to perform a self-absorption experiment[3] in order to measure the resonance scattering cross section. The coincidence rate was observed as a function of angle as an aluminum and a magnesium absorber were alternately inserted between the source and the resonance scattering detector. In this way it was possible to calculate the selective attenuation of the resonance radiation by the magnesium absorber. The ratio of the coincidence rates on resonance ($\theta = 120°$) to off resonance ($\theta = 114°$) for a magnesium absorber of 1.96 cm thickness was found to be 1.39 ± 0.26. For an aluminum absorber which attenuated the off-resonance coincidence rate by an amount similar to the magnesium absorber, the ratio of the coincidence rates at 120° and 114° was found to be 2.03 ± 0.3. The level width then follows from the attenuation upon inclusion of the dependence of the resonance effect on the spins involved and the thermal Doppler width. Preliminary data indicate a level width of 7×10^{-4} ev. This corresponds to a mean life of $\tau_\gamma = 0.95 \times 10^{-12}$ sec for the 1.38-Mev level. The statistical uncertainty in these measurements is about 90%. Helm[4] has estimated the mean life of this level from electron scattering data. His value, $\tau_\gamma = 1.9 \times 10^{-12}$ sec, is probably correct to within one order of magnitude and agrees well with the present work.

Experiments are continuing in order to improve the precision of the data and to study the apparent absence of beta recoil effects. A complete report will be published as soon as the measurements are concluded.

It is a pleasure to acknowledge our indebtedness to R. R. Lewis for many helpful discussions.

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POLARIZED NUCLEONS FROM THE PHOTODISINTEGRATION OF THE DEUTERON

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In a series of previous publications the polarization of the nucleons from the $D(\gamma,n)p$ reaction was calculated.[1,2] It was shown that, for unpolarized γ rays, the polarization $P(\theta)$ is rather sensitive to the final state interactions. The most important transitions are, in order of importance, $E1(^3S_1 + ^3D_1 \rightarrow ^3P_{0,1,2} + ^3F_2)$, $M1$ spin flip ($^3S_1 \rightarrow ^1S_0$ and $^3D_1 \rightarrow ^1D_2$), and $E2(^3S_1 + ^3D_1 \rightarrow ^3S_1 + ^3D_{1,2,3} + ^3G_3)$. The $E1$ transition amplitudes are rather well known from the analysis of the angular distribution and total cross section.[3,4] The forthcoming experiments on $P(\theta)$ should provide most important information on the relatively less known $M1$-transition amplitudes. The lack of symmetry of $P(\theta)$ about 90° provides addi-

tional information on the spin-dependent n-p forces.

The rather impressive success of the two-nucleon potential of Signell and Marshak[5] was recently extended by De Swart and Marshak[3] to the photodisintegration of the deuteron. It was shown that agreement with the experimental data in the medium energy range could be obtained without renouncing the Siegert theorem by assuming as the final state interaction the Signell-Marshak potential and assuming for the deuteron about 7% D-state probability. This high-percentage D state is not in contradiction with the observed magnetic moment and quadrupole moment of the deuteron. It was the aim of the present note to study $P(\theta)$ under these very same assumptions.

The angular distribution in the photodisintegration can be written as[2]

$$d\sigma/d\Omega = a(1 \pm \beta_1 \cos\theta) + b \sin^2\theta(1 \pm \beta_2 \cos\theta). \qquad (1)$$

The + sign (- sign) refers to the proton (neutron). Here θ is the angle between the direction $\vec{k}/k$ of the observed particle and the direction $\vec{\kappa}/\kappa$ of the incoming γ ray in the center-of-mass system, $\vec{\kappa}$ is the relative momentum of the γ ray, and $\vec{k}$ is the relative momentum of the observed particle. The polarization $P(\theta)$ below refers to the $\vec{n} = \vec{\kappa} \times \vec{k} / |\vec{\kappa} \times \vec{k}|$ direction.[6] We can write[2]

$$(d\sigma/d\Omega)P(\theta) = \sin\theta[\gamma_0 + \gamma_1 \cos\theta + \gamma_2 \cos^2\theta], \qquad (2)$$

where the γ_i's can be directly expressed in terms of the transition amplitudes and the phase shifts. There is a difference between the proton and neutron case so that we must distinguish between $\gamma_i(p)$ and $\gamma_i(n)$. The coefficients a, b, β_1, β_2, and γ_i were calculated numerically[4] using the Gartenhaus wave function[7] for the deuteron and the S.M. potential for the final state interactions. We have calculated the $E1$-$E1$, $E1$-$E2$, $E1$-$M1$ spin flip and $E2$-$M1$ spin flip interference terms, taking the tensor coupling of the final states exactly into account. The results are given in Table I. The proton polarization at γ-ray energies in the laboratory of 9.3 Mev, 22.5 Mev, and 80.4 Mev are shown in Fig. 1.

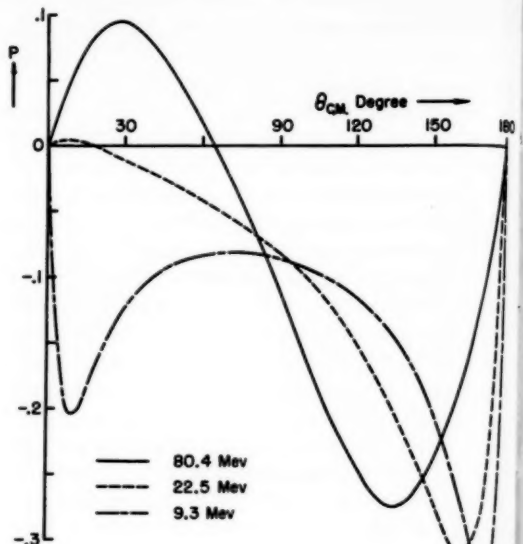

FIG. 1. Polarization of the protons for γ-ray energies in the lab of 9.3 Mev, 22.5 Mev, and 80.4 Mev.

At low energies the polarization is chiefly determined by the large γ_0 and the large b. This results in the characteristic saddle shape with two maxima at about 10° and about 170°. The polarization is negative.[8] The forward maximum

Table I. Summary of results. a, b, and the γ_i's are in μb/sterad.

E_γ lab (Mev)	9.3	11.3	22.5	40.6	53.7	80.4
a	4.45	4.22	4.46	4.89	4.87	4.54
b	171.5	135.7	49.3	16.7	9.39	4.35
β_1	0.04	0.05	0.10	0.13	0.13	0.14
β_2	0.18	0.20	0.32	0.47	0.57	0.70
$\gamma_0(p)$	-15.4	-11.6	-4.46	-2.18	-1.62	-1.02
$\gamma_0(n)$	-15.2	-11.5	-4.42	-1.87	-1.27	-0.58
$\gamma_1(p)$	2.16	2.8	3.87	3.38	2.84	1.97
$\gamma_1(n)$	4.61	4.9	4.68	3.48	2.65	1.84
$\gamma_2(p)$	0.43	0.43	0.86	0.79	0.66	0.80
$\gamma_2(n)$	-0.43	-0.43	-0.86	-0.79	-0.66	-0.80

decreases rapidly as the energy increases due to the decrease of γ_0. At about 20 Mev γ_0 and γ_1 are of the same order of magnitude and the forward maximum has totally disappeared. As the energy increases still more, γ_1 becomes larger than γ_0 and at the same time γ_2 comes into play. We get again a forward maximum. The forward polarization, however, is now positive. The backward maximum decreases slightly and shifts to somewhat smaller angles as the energy increases. In Fig. 2 the polarization of the proton as well as of the neutron is given for 80.4-Mev γ rays.

The difference between the proton and neutron polarization at lower energies is mainly due to the $M1$-$E2$ interference and at higher energies mainly to the $E1$-$E2$ interference. The neutron polarization is more negative (less positive) at small angles and less negative at large angles. The maximum difference occurs near the maxima of the polarization and is 2% at 9.3 Mev, 5% at 22.4 Mev, and 7% at 80.4 Mev.

We would like to thank Dr. P. S. Signell, Dr. P. J. Eberlein, and Mr. R. A. Bryan for their help with the numerical computations. J. J. deS. wishes to thank Professor R. E. Marshak for his guidance and stimulation in many discussions during the course of this work, and W. C. wishes to thank Professor N. Bohr and Professor A. Bohr for hospitality at the Institute for Theoretical Physics, University of Copenhagen, Denmark.

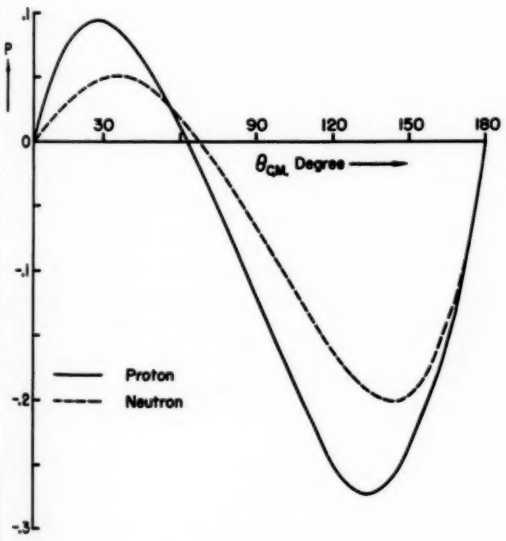

FIG. 2. Neutron and proton polarization at γ-ray energy in the lab of 80.4 Mev.

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[8]The signature of $P(\theta)$ is opposite to the present one if the convention $\vec{n}=\vec{k}\times\vec{\pi}/|\vec{k}\times\vec{\pi}|$ of reference 1 is used.

ELECTRON DECAY OF THE POSITIVE PION*

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In a previous communication[1] from this laboratory an unsuccessful search for the electronic decay mode of the pion was reported which made it seem unlikely that the branching ratio $f=(\pi\to e+\nu)/(\pi\to\mu+\nu)$ could be much larger than 2×10^{-5}. The remarkable success of the A-V theory[2,3] in accounting correctly for many related phenomena had raised doubt about our failure to observe this process. Accordingly, we decided to make a new attempt, and this effort was accelerated when we learned of the successful observation of the electronic mode by the CERN group[4] and by Steinberger.[5]

Our new work confirms the existence of the π-e mode in an amount not very different from the value 1.28×10^{-4} predicted by the universal A-V theory (without radiation correction).[6] We also know that additional confirmations have been obtained from experiments similar to ours which are in progress at Stanford[7] and Berkeley.[8] We

report here our evidence and our present best estimate of the value of the branching ratio.

We used the same method and much of the same apparatus as in the previous experiment but incorporated a few improvements which served to increase the ease and certainty of detecting the electrons from π-decay. The same double-focussing magnetic spectrometer was used to select the π-electrons of energy $E_\pi = 69.8$ Mev and to distinguish them from those originating from μ-decay which have maximum energy $E_\mu = 52.9$ Mev. It was modified to accommodate three detectors, admitting thereby simultaneous measurements in three adjacent momentum channels.

Pions of energy 70 Mev entered our spectrometer through counters 1 and 2, were moderated in polyethylene, and came to rest in the source counter 3. Electrons emerging from counter 3 with the proper energy and direction were focussed to traverse one of the counters 4*a*, 4*b*, or 4*c* and counter 5. The entering pions (Fig. 1) produced a coincidence (2,3) while the electron leaving counter 3 and arriving at counter 5 produced a coincidence $(\overline{2},3,5)$. The anticoincidence $\overline{2}$ reduced accidentals between a pion pulse in counter 3 and a spurious pulse in counters 4 and 5. The pion pulse from (2,3) was 50 mμsec long and was delayed 100 mμsec to make a coincidence with the 150-mμsec-long output of $(\overline{2},3,5)$. There were three final circuits which signalled the delayed coincidence between the pion (2,3) and its electron $(\overline{2},3,5)$ together with the fact that the electron also traversed one of the counters 4*a*, 4*b*, or 4*c*. An output from any one of these circuits was recorded on an appropriate scaler, triggered the sweep of a traveling wave oscilloscope, and stopped the count until it was

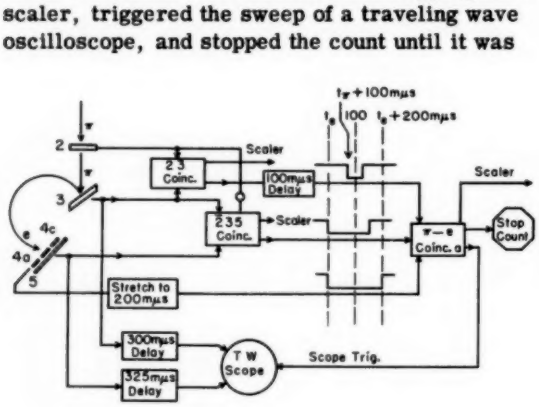

FIG. 1. Arrangement of electronics for detection of π-e events. Connections are shown for channel *a* and are duplicated for channels *b* and *c*.

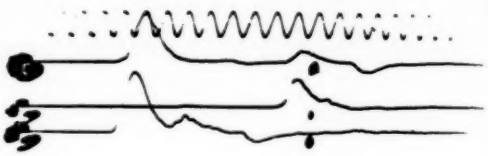

FIG. 2. Oscilloscope traces showing two π-e events. The middle trace is due to an accidental coincidence. Above the traces a 100-Mc/sec sine wave has been recorded to provide a timing scale.

manually reset. The oscilloscope was connected with sufficient delay directly to the anodes of the photomultipliers which looked at scintillators 3 and 5. The signature of a π-e event appeared as a large positive-going pulse due to the stopping pion in counter 3, a second but smaller positive pulse due to the electron leaving counter 3, this being followed by a small negative pulse due to the electron arriving at counter 5. The delays fixed the timing between the final (+,-) pair to be 32 mμsec, the timing of the first two positive pulses was used to verify the pion lifetime. Two signatures of this kind are shown in Fig. 2.

In order to make reliable pulse height and timing measurements, we limited our acceptance of good events to those in which the π-e decay time was greater than 24 mμsec but less than 150 mμsec. The choice of 24 mμsec was made to allow ample time for the anticoincidence pulse to decay and to assure a clear separation between the tail of the pion pulse and the following electron pulse in counter 3. The 150-mμsec upper limit was imposed by the length of the gates in the electronic circuitry. Other criteria which helped to discriminate true π-e events from accidentals were established by setting the spectrometer at 450 amperes (40 Mev/c) and studying the traces produced by the electrons from μ-decay. This study showed that all but 13% of the triggers were included when we required (a) electron pulse in counter 3 to be greater than 0.2 mm; (b) electron pulse in counter 5 to be greater than 0.2 mm but less than 1.0 mm; (c) timing between the final (+,-) pair to be more than 28 but less than 35 mμsec.

The branching ratio was obtained by comparing the integrals, with respect to ln$E$, of the π-electron and μ-electron parts of the spectrum. For the π-electrons there were 50 runs in which there was no obvious misbehavior. These runs gave us 950 triggers of which 159 passed our criteria for

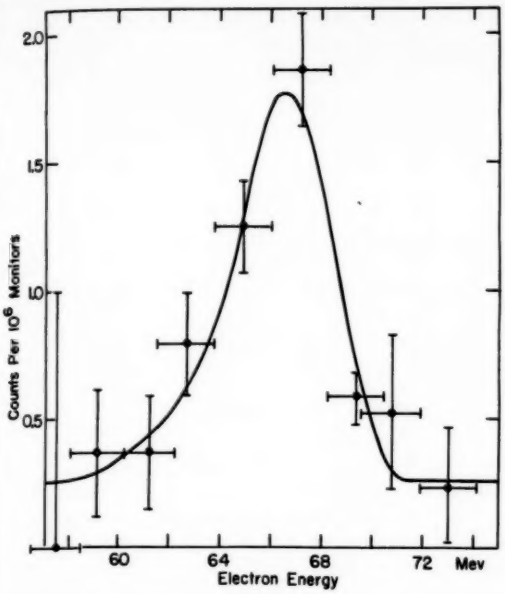

FIG. 3. Energy distribution of π-e events. Curve is best fit calculated by folding in the energy loss in source and the resolution of spectrometer.

a π-e event. These are displayed in Fig. 3. The smooth curve is the best fit calculated by folding into a δ-function at $E = E_\pi$ the known energy loss (including bremsstrahlung effects) in the source, as well as the resolution function of the spectrometer, known from α-particle calibrations, in the manner described previously.[1] A constant background entered as a parameter in the fit. This accounted for 39 ± 15 of our events. We could show from timing measurements that it was likely that 20 ± 5 of these were due to π-μ events in counter 3, in accidental coincidence with spurious counts in (4, 5). We supposed that the remainder were due to μ-electrons which managed, by scattering, to give counts (3, 4, 5) in accidental coincidence with pion counts in (2, 3). Taking account of the time distribution to be expected from such a composition of background events the mean life of the pion obtained from our π-e events turned out to be 25 ± 3 mμsec, close to the accepted value.

In recording the μ-electrons at the lower current settings of the spectrometer, we removed the requirement of a delayed pion coincidence, using simply the coincidence $(\bar{2}, 3, 5)$ with one of the counters 4. The data shown in Fig. 4 are from channel b (others were hardly different). It

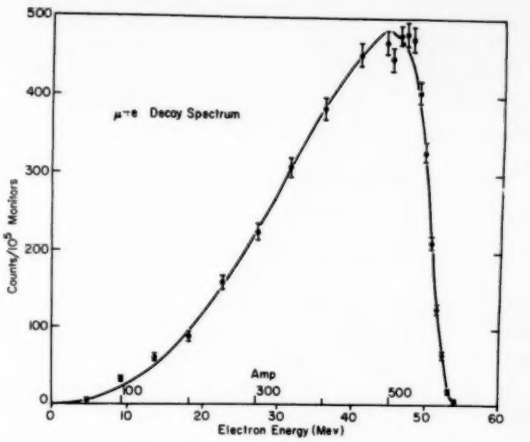

FIG. 4. Energy distribution of μ-e events in channel b. Curve is calculated best fit.

was fitted by adjusting the amplitude, the Michel parameter ρ, and the end-point energy E_μ. From all the data we found $E_\pi/E_\mu = 1.32 \pm 0.01$ as expected, and $\rho = 0.74 \pm 0.03$.

The ratio of the integrals was found to be $(3.49 \pm 0.56) \times 10^{-5}$. This must be multiplied by a factor 2.57 to take into account the pions which decayed outside the measured time interval, a factor 1.13 ± 0.01 for the π-e events missed by our selection criteria, a factor 1.04 ± 0.02 for losses due to bremsstrahlung, and a factor 0.98 ± 0.02 for the muons which escaped from the source. The branching ratio calculated from these data is $f = (1.03 \pm 0.20) \times 10^{-4}$. We have augmented the stated error a small amount to reflect our own uncertainty in the manner of dealing with the background.

Note: See ADDENDUM on page 64.

*Research supported by a joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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ASYMMETRY PARAMETER IN MUON DECAY*

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Increasingly convincing data have established the primary interaction in the weak interactions as V-A. Although it is known that $|C_V| \neq |C_A|$ in β decay, a fact generally attributed to meson cloud effects, no such renormalization effects are expected in muon decay. A precise measurement of the muon decay asymmetry would constitute a sensitive test of the theory, but such a measurement has heretofore been frustrated by the unknown polarizations of cyclotron extracted muon beams. In this experiment, the use of muons emitted near the cutoff angle from π^+ decays in flight results in an accurate knowledge of their polarization; and hence in a much smaller uncertainty in the asymmetry parameter than could be obtained in the original experiment of this kind.[1]

The arrangement used to define the transversely polarized muon beam is shown in Fig. 1. The energy of the incident pions and the muon energy interval defined by the thickness of the bromoform target determine the polarization of the stopped muons. The angular and energy distributions of the muons were studied to verify that the beam had the properties predicted by the

FIG. 1. The bromoform target is placed about two feet from the π^+ beam. Two counters, No. 2 ($2\frac{1}{2}$ in. $\times 5$ in. $\times 1/4$ in.) and No. 3 (3 in. $\times$ 6 in. $\times$ 1/4 in.), define muons emitted within 0.75° on each side of the cutoff angle. Counter No. 1 (5 in. $\times$ 5 in. $\times \frac{1}{2}$ in.) is placed in the π^+ beam. A set of Helmholtz coils is used to reduce the 15-gauss cyclotron fringing field to less than 0.5 gauss.

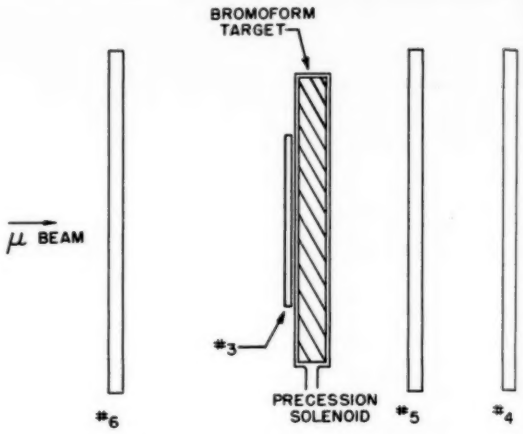

FIG. 2. The target is a Teflon container, 10 in. $\times$ 10 in. $\times$ 1 in., filled with bromoform. Two sets of counters are used to detect the electrons. They are No. 3 and No. 6, in anticoincidence with No. 5 (referred to as telescope 1); and No. 4 and No. 5, in anticoincidence with No. 3 and No. 6 (referred to as telescope 2). Counters No. 4, No. 5, and No. 6 are 12 in. $\times$ 12 in. $\times \frac{1}{2}$ in. The target is wound with an eleven-turn solenoid used to obtain the $\pm 90°$ precession.

kinematics. The setup for counting the μ^- decay electrons is shown in Fig. 2. A stopped μ signature, $123\bar{5}$, triggers a pulsed magnetic field of 1 μsec duration which is obtained from the discharge of a condenser through a solenoid wound around the target. After a 1.5-μsec delay, this count also opens a 3-μsec gate during which electrons are counted in telescope 1 and telescope 2. The pulsed magnetic field is used to cause a precession of the transversely polarized muons through $\pm 90°$, first so that the spins point into one telescope, then away from it, in one-hour cycles repeated fifty times. The magnetic field was calibrated by placing the target in the direct muon beam and observing the angular distribution of electrons from the decays of stopped muons.

The electron anisotropy observed with this apparatus is of the form $1+Ka\cos\theta$; where a is the muon decay asymmetry parameter averaged over the electron energy spectrum, θ is the angle between the muon spin and the axis of the detecting telescope; and K is a factor which depends on the polarization of the stopped muons, the energy selection made by the electron detector, and the extent of the target and electron counters. About 5000 events were obtained in

each telescope, and 9% of that number were measured to be accidental counts and counts resulting from mesons stopping in the target walls and in the precession solenoid. Correcting only for this small background, we obtain for the asymmetry in each telescope

$$Ka = 0.274 \pm 0.016 \quad \text{for telescope 1}$$

$$= 0.276 \pm 0.016 \quad \text{for telescope 2,}$$

where the uncertainties indicated are statistical only.

The factors K for the polarization and the counting arrangement in this experiment are:

$$K = 0.850 \pm 0.024 \quad \text{for telescope 1}$$

$$= 0.844 \pm 0.024 \quad \text{for telescope 2.}$$

To compute K, it was necessary to know the fraction of electrons which has insufficient energy to be counted in the detecting telescopes. The energy and angular dependence of the muon decay electrons was assumed to be the one predicted by the two-component neutrino theory.[2] The probability of counting an electron as a function of energy and material traversed was obtained with Wilson's range formula,[3] and the measured distribution of muons stopping in the target. The errors indicated in K are due primarily to a 2% uncertainty in the polarization resulting from the uncertainty in the muon energy interval defined by the bromoform target, and to a 2% uncertainty which is contributed by the energy dependence of the electron counting efficiency; and also to an additional 1% uncertainty due to the fact that the current required to obtain the $\pm 90°$ precession is known to 10%.

Combining the data from the two telescopes, the asymmetry parameter is found to be

$$|a| \geq 0.325 \pm 0.015,$$

where the inequality arises from the possible depolarization of the muons stopping in bromoform. These depolarization effects are unknown; but, of a large number of materials tested, bromoform has been shown[4] to cause the least depolarization.

With the assumption of two-component neutrino theory, and with the coupling scheme $(e\nu)(\nu\mu)$,

$$3|a| = |\xi| = \left| \frac{|g_S|^2 - 4|g_V|^2}{|g_S|^2 + 4|g_V|^2} \right|;$$

then we have shown that

$$|\xi| \geq 0.97 \pm 0.05.$$

Measurements of the μ spin direction[5] indicate that ξ is negative, so that we have, in terms of the coupling constants,

$$|g_S|^2/|g_V|^2 \leq 0.05 \pm 0.09.$$

*Research supported in part by the Office of Naval Research and the U. S. Atomic Energy Commission.

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POSSIBLE DETERMINATION OF $K\Lambda$ RELATIVE PARITY*

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(Received December 16, 1958)

Okun' and Pomeranchuk[1] have suggested that a study of the absorption of bound K^- mesons in hydrogen according to the infrequently occurring reactions

$$K^- + p \rightarrow \Lambda^0 + \pi^0 + \pi^0, \quad (1)$$

$$K^- + p \rightarrow \Lambda^0 + \pi^+ + \pi^-, \quad (2)$$

might yield information regarding $P_{K\Lambda}$, the $K\Lambda$ relative parity. The K^- meson was assumed to be initially in an S state. For reaction (2) it was found that an angular correlation between the directions of the relative momentum $\vec{p}_\pi$ of the two pions and the momentum $\vec{p}_\Lambda$ of the Λ^0 hyperon would indicate $P_{K\Lambda} = -1$. Also, $P_{K\Lambda} = -1$ would result in the Λ^0 from reaction (2) being polarized with respect to the reaction plane. In addition, the probability for reaction (1) relative to that for reaction (2) depends on $P_{K\Lambda}$. However, as Okun' and Pomeranchuk have emphasized, the interpretation of experimental data

would be ambiguous due to our present lack of knowledge regarding the P wave interaction in the KN system and the time dependence of the relative populations of levels in newly formed K^- mesonic atoms.

The purpose of this note is to suggest that much the same results obtain for the absorption in flight of K^- mesons with laboratory energies below approximately 30 Mev. Moreover, other effects will be present which also depend on $P_{K\Lambda}$. If $P_{K\Lambda}=-1$ the directions of both $\vec{p}_\pi$ and $\vec{p}_\Lambda$ in the center-of-mass (c.m.) system will be correlated in reaction (2) with the direction of $\vec{p}_K$, the c.m. momentum of the incident K^- meson; in reaction (1) only $\vec{p}_\Lambda$ and $\vec{p}_K$ will have an angular correlation. For $P_{K\Lambda}=-1$ the Λ^0 will be polarized in both (1) and (2). Furthermore, $P_{K\Lambda}$ determines the energy dependence of the cross sections.

The physical reasons for these assertions are as follows. In the c.m. system the initial relative motion of the K^- meson and proton is described by a superposition of partial waves, each of which has a definite total angular momentum and parity which are both conserved in the reactions (1) and (2). It is possible to transform the c.m. coordinates and momenta describing the products of these reactions in such a way that the total angular momentum operator is given by the sum of two angular momentum operators which are related to the motion of the Λ^0 and the relative motion of the two pions, respectively. The final state which corresponds to a particular incident partial wave may then be represented asymptotically by an infinite series of products of two wave functions, each product describing a suitable combination of spin and orbital motion of the Λ^0 and the two pions which is compatible with the conservation laws and possible identity of the pions. For these final states it will be assumed that only the lowest possible combinations of orbital angular momenta are significant for the range of K^- energies being considered. This assumption is based on the fact that the energies released in (1) and (2) are low, being 47 and 38 Mev, respectively, for capture at rest. The wave functions for higher orbital angular momenta will have much smaller amplitudes in the region of interaction, which is assumed to be less than 10^{-13} cm. These considerations are similar to those employed by Dalitz[2] in his analysis of τ decay. The spins of the K^- and Λ^0 are assumed to be 0 and $\frac{1}{2}$, respectively. If $P_{K\Lambda}=+1$ an incident S wave is transformed into two final S waves. The effect of incident P waves will be small. The angular distributions will be isotropic, and angular momentum conservation will require the Λ^0 to be unpolarized. However, if $P_{K\Lambda}=-1$ an incident S wave will emerge as suitable combination of S and P waves, and, in the same order of approximation, an incident $P_{1/2}$ wave will emerge as a combination of two S waves. The angular distributions will not be isotropic, and, due to interference effects, the Λ^0 will be polarized. There will be a different energy dependence of the cross sections because the amplitudes of the P waves in the interaction region increase as the K^- energy becomes larger.

These qualitative considerations will now be put in a more definite form. The amplitudes for the transitions are expanded in powers of $(\vec{p}_K R)$, $(\vec{p}_\pi R)$, and $(\vec{p}_\Lambda R)$ where R is a distance determined by the range of the interaction. For the energies being considered it will suffice to retain only the terms of lowest order. It will be assumed that reactions (1) and (2) conserve isotopic spin. The amplitudes A_1 and A_2 for reactions (1) and (2), respectively, are scalar (pseudoscalar) if $P_{K\Lambda}=+1$ ($P_{K\Lambda}=-1$). In the following $\vec{\sigma}$ is the vector of the Pauli matrices, $d\rho_F$ denotes the density of final states, $\vec{\zeta}$ is a unit pseudovector in the direction of polarization, v is the velocity of the incident K^- meson, and Q_1 and Q_2 are the energies released in (1) and (2), respectively, for capture at rest. The total cross sections σ_1 and σ_2 for (1) and (2), respectively, obtained by integrating the differential cross sections $d\sigma_1$ and $d\sigma_2$ over the available phase space, are also given below. Also, T_i is given by

$$T_i = Q_i + p_K^2/(2\mu), \quad i = 1,2$$

where μ is the reduced mass of the Kp system.

For $P_{K\Lambda}=+1$ we have

$$A_1 = -a'/\sqrt{2}, \quad A_2 = a',$$

$$d\sigma_1 = (1/4v)\,|a'|^2\,d\rho_F,$$

$$d\sigma_2 = (1/2v)\,|a'|^2\,d\rho_F,$$

$$\sigma_1 \sim (1/2v)\,T_1^2\,|a'|^2,$$

$$\sigma_2 \sim (1/v)\,T_2^2\,|a'|^2.$$

For $P_{K\Lambda}=-1$ we have

$$A_1 = -(a\vec{\sigma}\cdot\vec{p}_K + b\vec{\sigma}\cdot\vec{p}_\Lambda)/\sqrt{2},$$

$$A_2 = a\vec{\sigma}\cdot\vec{p}_K + b\vec{\sigma}\cdot\vec{p}_\Lambda + c\vec{\sigma}\cdot\vec{p}_\pi,$$

$$d\sigma_1 = (1/4v)\{|a|^2 p_K{}^2 + |b|^2 p_\Lambda{}^2 + 2\,\mathrm{Re}\,(a^*b)\vec{p}_K\cdot\vec{p}_\Lambda + 2\,\mathrm{Im}\,(a^*b)(\vec{p}_K\times\vec{p}_\Lambda)\cdot\vec{\xi}\}d\rho_F,$$

$$d\sigma_2 = (1/2v)\{|a|^2 p_K{}^2 + |b|^2 p_\Lambda{}^2 + |c|^2 p_\pi{}^2 + 2\,\mathrm{Re}\,[a^*b\vec{p}_K\cdot\vec{p}_\Lambda + b^*c\vec{p}_\Lambda\cdot\vec{p}_\pi + c^*a\vec{p}_\pi\cdot\vec{p}_K] + 2\,\mathrm{Im}\,[a^*b(\vec{p}_K\times\vec{p}_\Lambda) + b^*c(\vec{p}_\Lambda\times\vec{p}_\pi) + c^*a(\vec{p}_\pi\times\vec{p}_K)]\cdot\vec{\xi}\}d\rho_F,$$

$$\sigma_1 \sim (1/2v)\,T_1{}^2\{0.34|b|^2(2\mu Q_1) + [|a|^2 + 0.34|b|^2]p_K{}^2\},$$

$$\sigma_2 \sim (1/v)\,T_2{}^2\{[0.34|b|^2 + 0.43|c|^2](2\mu Q_2) + [|a|^2 + 0.34|b|^2 + 0.43|c|^2]p_K{}^2\}.$$

The interpretation of data on absorption of K^- mesons in flight at low energies should be less ambiguous than that for absorption at rest. The greatest difficulty would appear to be the small cross sections for reactions (1) and (2).

*Supported by the National Science Foundation.

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[2]See, for example, R. H. Dalitz, Reports on Progress in Physics (the Physical Society, London, 1957), Vol. 20, p. 163, and references contained therein.

POSSIBLE EXPLANATION OF HYPERFRAGMENT SUPPRESSION IN K^--d REACTIONS*

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(Received December 15, 1958)

One of the reactions that can occur when a K^- meson is captured from an atomic orbit about a deuteron is

$$K^- + d \rightarrow \Sigma^- + n + \pi^+. \qquad (1)$$

Pais and Treiman[1] have investigated the probability of formation of a bound state of the (Σ^-, n) system, if such a system exists. They concluded that if the (Σ^-, n) binding energy was of the order of a few tenths of a Mev or larger, the probability of bound-state formation via reaction (1) is ~40%, except if the K^- meson were pseudoscalar, and the singlet state of the (Σ^-, n) system is the only bound system.

A re-examination of this problem indicates that there is one important process that was not included in their work; that is, the absorption of the K^- meson from the $2P$ atomic orbit via an s-wave capture process of the type

$$K^- + p \rightarrow \Sigma^- + \pi^+. \qquad (2)$$

The rate of this (K^-, p) s-wave capture from the $2P$ orbit is nonvanishing since the deuteron has a finite size and the rate is proportional to the square of the $2P$ orbital wave function evaluated at the position of the proton in the deuteron. Furthermore, a crude estimate of the total absorption rate from the $2P$ orbit by s-wave capture $\Gamma(P,s)$, indicates that it is ~10 times larger than either the $2P \rightarrow 1S$ radiative rate $\Gamma(P,\gamma)$ or the $2P$ absorption rate via p-wave capture. It is the purpose of this letter to show that the probability of the formation of the (Σ^-, n) bound state via the s-wave capture process from the $2P$ orbit is one order of magnitude smaller than for the processes considered in PT.

In the following discussion, the notation and approximations of PT are used. These approximations consist in the use of the impulse approximation and two-parameter Hulthén wave functions for the bound states.

Assuming that the K^- meson is pseudoscalar with respect to the (Σ^-, n) system, and that the (Σ^-, n) bound state has spin 1, then, analogous to Eq. (8) of PT, the bound-state transition rate via the s-wave (K^-, p) interaction can be written

$$R_B = \tfrac{2}{3}\pi|a|^2\sum_{m_l}\int\left|y^{m_l}\right|^2\frac{d\vec{P}_\pi}{(2\pi)^3\,dE}, \qquad (3)$$

where

$$y^{m_l} = \int d\vec{r}\; v_B{}^*(\vec{r})e^{-i\vec{k}_0\cdot\vec{r}}u(\vec{r})\,\psi_{2P}{}^{m_l}(\vec{r}/2). \qquad (4)$$

ψ_{2P} is the K^- wave function in the $2P$ orbit and the other symbols are explained in PT. Since only small values of the argument of ψ_{2P} are important, $\psi_{2P}(\vec{r}/2)$ can be replaced by $\frac{1}{2}\vec{r}\cdot\nabla\psi_{2P}(0)$, and the evaluation of R_B is straightforward in

terms of the parameters of the bound-state wave functions.

Again in a manner exactly analogous to Eq. (10) of PT, closure is invoked to get for the total transition rate for production into the bound and free states

$$R_F+R_B \cong \tfrac{2}{3}\pi|a|^2\sum_{m_l}\int I^{m_l}\frac{d\vec{P}_\pi}{(2\pi)^3\,dE}, \qquad (5)$$

where

$$I^{m_l}=\int d\vec{r}\,|u(\vec{r})|^2|\psi_{2P}{}^{m_l}(\vec{r}/2)|^2. \qquad (6)$$

The ratio $R_B/(R_B+R_F)$ is then easily computed. (The small variation of the pion phase space factor in free production as compared to its value in bound production is ignored.)

The values of the ratio $R_B/(R_B+R_F)$ are plotted in Fig. 1 for several values of the (Σ^-,n) binding energy, ϵ', and for two values of the effective range ρ'. Instead of getting values ~40% for this ratio as found by PT, we find values ~6%. Hence, in the most favorable case of a pseudoscalar K^- meson and a triplet (Σ^-,n) bound state, the ratio of (Σ^-,n) bound to (Σ^-+n) free via s-state capture from the $2P$ orbit is only ~6%. If the (Σ^-,n) bound state has spin 0, then this mode of capture gives zero probability for bound-state formation for a pseudoscalar K^- meson. For scalar K^- mesons, and a (Σ^-,n) bound state with spin 0 or 1, the bound-state formation probability will be less than but of the same order of magnitude as the probabilities plotted in Fig. 1 (see discussion in PT).

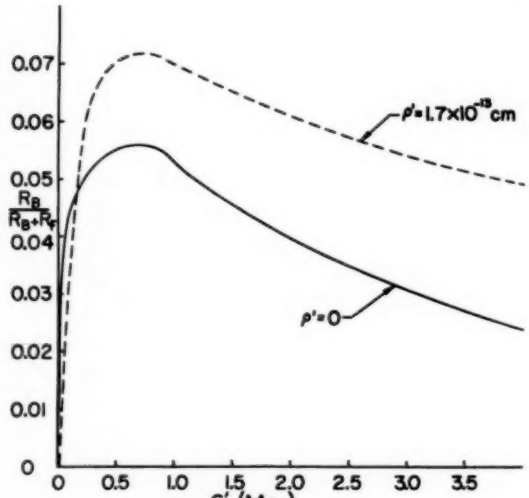

FIG. 1. The ratio of (Σ^-,n) bound-state formation rate to total capture rate from the $2P$ orbit of the (K^-,d) atom via s-wave capture, is plotted as a function of the (Σ^-,n) binding energy, ϵ', for two values of the (Σ^-,n) effective range, ρ'.

The results of the above calculation are significant for the over-all probability of (Σ^-,n) bound-state formation if $\Gamma(P,s)$ dominates the capture process. The much higher values of $R_B/(R_B+R_F)$ as found by PT are still applicable for s-capture from an nS orbit or for p-capture from the $2P$ orbit. (The probability that a K^- meson reach an nS orbit with $n>1$, before reaching the $2P$ orbit is believed to be quite small since the original atomic capture favors high n and consequently high l.) A rough estimate of the absolute magnitude of $\Gamma(P,s)$ can be obtained by using the data presented by Tripp[2] on the cross sections for (K^-,p) absorptions in flight at low energies. Assuming, as is suggested by the energy dependence of the cross sections, that the observed cross sections are mostly s-wave for $P_{K^-}\lesssim 240$ Mev/c, one deduces for the reaction of Eq. (2),

$$2\pi|a|^2\int\frac{d\vec{P}_\pi}{(2\pi)^3\,dE}\cong v_{K^-}p\sigma_{\Sigma^-\pi^+}$$
$$=23.4\times10^{-17}\,\mathrm{cm}^3/\mathrm{sec}. \qquad (7)$$

One then finds

$$R_F+R_B\cong 1.82\times10^{12}\,\mathrm{sec}^{-1}. \qquad (8)$$

Hence, $\Gamma(P,s)\gtrsim 5\times10^{12}\,\mathrm{sec}^{-1}$, when one includes the other (K^-,p) capture reactions plus the (K^-,n) s-wave captures. On the other hand, $\Gamma(P,\gamma)=4.78\times10^{11}\,\mathrm{sec}^{-1}$, so that $\Gamma(P,\gamma)/\Gamma(P,s)\lesssim(1/10)$. This estimate implies that the K^- meson is captured $\gtrsim$90% of the time from the $2P$ orbit before it has a chance to radiate down to the $1S$ orbit.

An estimate of the rate of p-wave capture out of the $2P$ orbit is much more difficult, since the absolute p-wave cross section in (K^-,p) interactions is not known. From analysis of the (K^-,p) cross sections in flight, Jackson et al.[3] have estimated that the capture rate for the reaction $K^-+p\to\Sigma^-+\pi^+$ out of the $2P$ orbit of the (K^-,p) atom is $\lesssim 1\times10^{11}\,\mathrm{sec}^{-1}$. This would imply that in deuterium the transition probability for the reaction $K^-+d\to\Sigma^-+\pi^++n$ via p-wave capture from the $2P$ orbit is $\lesssim 2.6\times10^{11}\,\mathrm{sec}^{-1}$. Comparing this estimate with Eq. (8), we find that, for the reaction $K^-+d\to\Sigma^-+n+\pi^+$, the p-wave capture rate

from the $2P$ orbit is smaller than the s-wave capture from the $2P$ orbit by a factor ≥ 7. Much more data on (K^-, p) collisions in flight will be necessary to make the above estimate reliable.

The non-spin-flip amplitude of the p-wave capture process can interfere with the s-wave capture process in the calculation of the probability of formation of the (Σ^-, n) compound. Unfortunately, not enough is known experimentally so as to determine whether this interference is constructive or destructive. Experimentally, Tripp[2] reports no (Σ^-, n) compounds out of 49 events of the type $K^- + d \rightarrow \Sigma^- + n + \pi^+$. The analysis presented here indicates that this experimental result does not as yet exclude the possibility that a (Σ^-, n) bound state exists. Hence a continued, intensive search for such a compound is indicated.

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*Supported in part by the U. S. Atomic Energy Commission, while one of the authors (G. A. S.) was in residence at the University of Wisconsin.

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[2]R. D. Tripp, 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), p. 184.

[3]Jackson, Ravenhall, and Wyld, Nuovo cimento 9, 834 (1958).

IONIZATION OF THE UPPER ATMOSPHERE BY LOW-ENERGY CHARGED PARTICLES FROM A SOLAR FLARE

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(Received November 25, 1958)

Excess ionization of the upper atmosphere at high latitudes following solar flares has been observed on several occasions during the International Geophysical Year, through studies of the absorption of cosmic radio noise. One such event, discussed in this note, occurred on July 29, 1958 following the class 3 solar flare at 03:03 UT.[1] The recordings of the 27.6-Mc/sec cosmic noise received on this date at Thule, Greenland and Barrow and College, Alaska are reproduced in Fig. 1. Identical receiving equipments, consisting of riometers[3] connected to wide beam, vertically-directed Yagi antennas, were used at each station. The dashed curve on each trace shows the strength of the cosmic

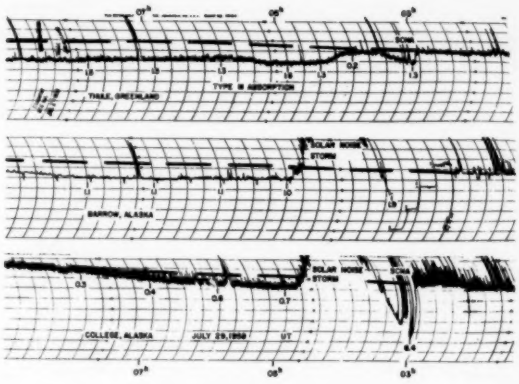

FIG. 1. Cosmic noise absorption on July 29, 1958. The dashed curves show the cosmic noise signal received under quiet conditions at each station; the computed values of absorption in decibels are shown for representative times. A constant amount of interference is present on the Barrow recording and has been included in the Barrow quiet-day curve (the actual cosmic noise level can be seen during the half-hour keying tags of the interfering transmitter).

noise signal that would have been received under undisturbed conditions. The computed values of absorption in decibels are also shown on the respective recordings.

The sudden cosmic noise absorption (SCNA) coincident with the visible flare is evident on the College and Thule recordings. The Barrow equipment was being calibrated at this time. The SCNA is known to be the result of ionization of the D region by the enhanced ultraviolet and x-ray flux from the flare. The difference in magnitude of the SCNA at College and Thule is a reflection of the greater solar zenith angle at the latter station ($\chi = 83.5°$ at Thule and $\chi = 73.5°$ at College during the peak of the SCNA).

A very intense 27.6-Mc/sec solar noise storm was recorded at College and Barrow which obscured the recovery of the SCNA at those stations. This noise storm was not seen at Thule, through a fortunate combination of large solar zenith angle and orientation of the antenna, which placed the sun in a null of the beam pattern. Thus it is possible to follow the recovery of the SCNA at Thule, and we find that the absorption had decreased to about 0.2 db by 04^h UT.

Approximately sixty-five minutes after the onset of the SCNA the absorption again increased at Thule. We interpret this increase as arising from ionization of the upper atmosphere by charged particles originating at the sun at the

time of the flare. On this assumption is is possible to obtain the penetration depth and magnetic cutoff latitude of the particles, assuming them to be protons. For example, using Bailey's Fig. 4,[3] we find that protons traveling in Störmer orbits from the sun to the earth in 65 minutes would have an anergy of 10 Mev, a penetration depth of about 67 km, and a magnetic cutoff latitude of 72 degrees.[4]

The inferred penetration depth is consistent with the observed absorption. The absorption in decibels is proportional to the product of the electron density and the electron collisional frequency, integrated throughout the absorbing region; it can be shown[5] that only a small number of incoming protons penetrating to between 60 and 70 km would be required to produce the observed absorption.

Both Barrow and College also show the post-SCNA absorption. The duration of the absorption at Barrow was similar to that at Thule; however, the absorption at College was weak and decreased rapidly. The riometer at Farewell, Alaska (250 miles southwest of College) showed no post-SCNA absorption. We therefore conclude that College was close to the cutoff latitude. In Table I we have given both the geomagnetic latitude and the dip-pole "latitude" of each of the stations. We have arbitrarily defined the latter quantity as the complement of the angular separation between the station and the north magnetic dip-pole. The presence of absorption at College, and the similarity of the event at Thule and Barrow with regard both to duration and magnitude, indicate that the particles were influenced by a field more nearly like the dip-pole field than the dipole field. This conclusion is consistent with the observation[6] that the cosmic ray equator corresponds more nearly to the dip equator than to the dipole equator.

Table I. Magnetic latitudes of IGY riometer stations.

Station	Geomagnetic latitude[a]	"Dip-pole" latitude[b]
Thule	88.0°	81.9°
Barrow	69.0°	74.8°
College	64.9°	71.8°
Farewell	61.6°	68.4°

[a]Coordinates of geomagnetic pole taken as 78.6°N, 70.6°W.

[b]Coordinates of dip-pole taken as 76°N, 102°W. A field concentric with the earth was assumed for these computations.

Two general approaches to the problem of the long duration of the post-flare absorption, seen in this event at both Barrow and Thule (at least 22 hours at both stations) are possible: (a) the absorption may be due to a continuing bombardment of the atmosphere by particles of the correct energies, or (b) the duration may be a consequence of unusually slow recombination processes in the absorbing region. Bailey[3] has used the latter approach in treating the February 23, 1956 event; however, the question still seems open at this time.

Three other examples of high-latitude absorption of cosmic noise following solar flares, called Type III absorption by Reid and Collins, have been published. Little and Leinbach[7] gave the College data for the February 23, 1956 event; Reid and Collins[5] discuss two similar events observed at Churchill, Manitoba in July and October, 1957.

We conclude that observations of the absorption of 27.6-Mc/sec cosmic radio noise can be used to detect the presence of low-energy particles originating at the sun during solar flares. The cosmic noise technique is most sensitive to those particles which penetrate to heights of between 90 and 50 km, corresponding to proton energies of the order of 1 to 30 Mev. Ionization by these particles can be observed at stations located at magnetic latitudes of greater than about 68 degrees.

The authors wish to express their thanks to the field station operators, Mr. A. R. Franzke at Barrow, Mr. Harvey Seabrook at Farewell, and the Air Force IGY team at Thule, for the successful operation of the IGY riometers. Establishment of the field stations has been made possible by the National Science Foundation through IGY contracts 6.20 and 1.43. One of us (H.L.) is indebted to Dr. C. G. Little for many helpful discussions of cosmic noise absorption.

[1]Preliminary Report of Solar Activity, TR-361, High Altitude Observatory, Boulder, Colorado.

[2]A description of the IGY riometers (relative ionospheric opacity meters) is scheduled for publication in the Proc. Inst. Radio Engrs. The cosmic noise technique has already been discussed. C. G. Little and H. Leinbach, Proc. Inst. Radio Engrs. 46, 334 (1958).

[3]D. K. Bailey, J. Geophys. Research 62, 431 (1957).

[4]We have not as yet given any detailed consideration to the possible existence of localized impact zones for

this particular event.

[5]G. C. Reid and C. Collins, J. Atmos and Terrest. Phys. (to be published).

[6]J. A. Simpson, Geophysical Monograph No. 2 (American Geophysical Union, Washington, D. C., 1958), pp. 65-70.

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γ_5-INVARIANCE AND STRONG INTERACTIONS

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The study of the weak interactions has shown that parity conservation is not a universal law. It rather seems that only the conservation of CP is strictly valid.[1] In addition there is now much evidence that the weak interactions possess a typical invariance, which is called the γ_5-invariance or chirality invariance. It states that the interaction Hamiltonian for weak interactions is invariant under the transformation $\psi \to \gamma_5 \psi$ for each spinor particle involved. This invariance automatically leads to a four-fermion interaction of the form V-A.[2-4] It should be pointed out that the electromagnetic interaction also has this chirality invariance and one may state that the nonexistence of a direct anomalous magnetic moment coupling of bare elementary particles with the electromagnetic field is a special consequence of this law.

The purpose of this note is to discuss the possibility of extending the γ_5-invariance to strong interactions. At first sight this seems impossible since parity conservation is well established for nucleon-nucleon and nucleon-pion interactions.[5-7] The γ_5-invariance requires the nucleon-pion interaction Lagrangian density to be of the form

$$\mathfrak{L}_{\text{int}} = \frac{f_\pi}{m_\pi} \tfrac{1}{2} \bar{\psi} i\gamma_\mu (1+\gamma_5) \vec{\tau}\psi \cdot \frac{\partial}{\partial x_\mu} \vec{\phi}. \tag{1}$$

The symbol ψ stands for the nucleon wave function, $\vec{\phi}$ for the π-meson field, and the arrows refer to isotopic spin components. It is easily seen, however, that the vector part of this interaction gives no contribution in zeroth order by virtue of the continuity equation and hence no effects of parity nonconservation can occur in this order.

This fact suggests a generalization of (1). Let us write the baryon-pion interaction Lagrangian density in the form

$$\mathfrak{L}_{\text{int}} = \frac{f_\pi}{m_\pi} \left(\vec{j}_\mu + \vec{j}_\mu{}^A \right) \cdot \frac{\partial}{\partial x_\mu} \vec{\phi}, \tag{2}$$

where $\vec{j}_\mu{}^A$ denotes an axial vector current. This interaction should be determined in such a way that $\vec{j}_\mu$ occurring in (2) is identical to the total isotopic spin vector current following from the Lagrangian and hence obeys the continuity equation

$$\frac{\partial}{\partial x_\mu} \vec{j}_\mu = 0. \tag{3}$$

The idea of a coupling with the total current is similar to the Feynman–Gell-Mann hypothesis that total currents are coupled in weak interactions.[4] Since in the present case, however, the current is obtained from the interaction itself, a "self-consistent" procedure is necessary. The explicit form for $\mathfrak{L}_{\text{int}}$ which fulfills the conditions (2) and (3) is

$$\mathfrak{L}_{\text{int}} = \frac{f_\pi}{m_\pi} Q\{\vec{j}_\mu{}^0 + \vec{j}_\mu{}^A\} \cdot \frac{\partial}{\partial x_\mu} \vec{\phi}. \tag{4}$$

The operator Q is defined through its operation on any isotopic-spin vector $\vec{X}$ by

$$Q\{\vec{X}\} = \frac{1}{1+(f_\pi/m_\pi)^2 \phi^2} \left(\vec{X} + \frac{f_\pi}{m_\pi} [\vec{\phi} \times \vec{X}] + \frac{f_\pi^2}{m_\pi^2} \vec{\phi} (\vec{\phi} \cdot \vec{X}) \right). \tag{5}$$

$\vec{j}_\mu{}^0$ denotes the isotopic spin vector current for free fields. The relative coefficients of its different parts are determined from the requirement of isotopic spin invariance of the Lagrangian.

$$\vec{j}_\mu{}^0 = \tfrac{1}{2} \bar{\psi} i\gamma_\mu \vec{\tau}\psi + [\vec{\Sigma} \times \gamma_\mu \vec{\Sigma}] + \tfrac{1}{2} \bar{\Xi} i\gamma_\mu \vec{\tau} \Xi + \pi\text{- and } K\text{-meson currents.} \tag{6}$$

The γ_5-invariance requirement which we want to impose on (4) determines now the axial vector current $\vec{j}_\mu{}^A$ uniquely. Thus the relative baryon-pion coupling constants are given by the coefficients appearing in (6).

The continuity equation which follows from the new Lagrangian automatically assures that the interaction Lagrangian density, Eq. (4), can be replaced by an effective interaction with a pure axial vector coupling. Hence parity remains

conserved in spite of the γ_5-invariant interaction from which we started. It can be shown[8] that even in the presence of electromagnetic interactions, where the isotopic vector current $\vec{j}_\mu$ is not divergenceless any longer, the effective interaction is the same as the one from the axial vector interaction alone.

Equation (6) and therefore the resulting effective interaction contains no $\Lambda\Sigma\pi$ coupling. This is due to the fact that the Λ particle has isotopic spin zero. If we assume, however, as Gell-Mann did,[9] that the mass difference between Λ and Σ is due to K interactions and that the pairs Σ^+, $(\Lambda-\Sigma^0)/\sqrt{2}$ and $(\Lambda+\Sigma^0)/\sqrt{2}$, Σ^- behave as isotopic spin doublets, Eq. (6) must be replaced by

$$\vec{j}_\mu^{\,0} = \tfrac{1}{2}\bar{\psi}i\gamma_\mu\vec{\tau}\psi + \tfrac{1}{2}[\vec{\bar{\Sigma}}\times\gamma_\mu\vec{\Sigma}] + \tfrac{1}{2}\bar{\Lambda}i\gamma_\mu\vec{\Sigma} + \tfrac{1}{2}\vec{\bar{\Sigma}}i\gamma_\mu\Lambda + \pi\text{- and } K\text{-meson currents.} \quad (7)$$

From the γ_5-invariance requirement, the current $\vec{j}_\mu^{\,A}$ which enters the effective interaction now possesses Gell-Mann's global symmetry[9]: all baryons have equal parity and the baryon-pion coupling constants are of equal magnitude.

The chirality invariance principle, therefore, can be extended to the baryon-pion interactions. Turning the arguments around, one may say that this principle together with the idea of the coupling of total currents leads to a parity-conserving baryon-pion interaction.

The consequence of the hypotheses of a universal γ_5-invariance for the K-meson interactions is less obvious. Contrary to the case of baryon-pion interactions there does not seem to exist a conserved (strangeness-carrying) current which would ultimately lead to a parity-conserving effective interaction. Therefore a breakdown of strict parity conservation in K-meson interactions can be expected. The experimental answer[10,11] to this question seems not yet conclusive though indications of a possible parity nonconservation in K-interactions have recently been reported.[12]

More detailed calculations and discussions on this subject will be published in Zeitschrift für Physik.[8]

The author wishes to thank Professor J. H. D. Jensen and Dr. G. Kramer for helpful discussions.

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[5]N. Tanner, Phys. Rev. 107, 1203 (1957).
[6]D. H. Wilkinson, Phys. Rev. 109, 1603 (1958).
[7]Heer, Roberts, and Tinlot, Phys. Rev. 111, 645 (1958).
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[9]M. Gell-Mann, Phys. Rev. 106, 1296 (1957).
[10]See, for instance, Blumenfeld, Chinowsky, and Lederman, Nuovo cimento 8, 196 (1958).
[11]Crawford, Cresti, Good, Solmitz, and Stevenson, Phys. Rev. Lett. 1, 209 (1958).
[12]L. M. Lederman (private communications).

ERRATUM

Li^7 AND F^{19} NUCLEAR MAGNETIC RESONANCES IN NEUTRON-IRRADIATED LiF. P. J. Ring, J. G. O'Keefe, and P. J. Bray [Phys. Rev. Lett. 1, 453 (1958)].

In the third paragraph, second sentence, "about 7% of the fluorine nuclei" is incorrect. It should read "about 2% of the fluorine nuclei."

ADDENDUM

ELECTRON DECAY OF THE POSITIVE PION. H. L. Anderson, T. Fujii, R. H. Miller, and L. Tau [Phys. Rev. Lett. 2, 53 (1959)].

S. M. Berman, Phys. Rev. Lett. 1, 468 (1958), has shown that due to radiative effects, the theoretical estimate of the branching ratio depends on the size of the energy interval below the maximum energy over which electron counts are accepted. In our case this is close to 1.1 Mev, the channel width indicated on the points of Fig. 3. With this, Berman's formula gives a branching ratio of 1.14×10^{-4}. Our result is $(10\pm18)\%$ lower.

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

SIMPLIFIED TREATMENT FOR STRONG SHORT-RANGE REPULSIONS IN N-PARTICLE SYSTEMS. I. GENERAL THEORY. John W. Clark and Eugene Feenberg, Wayman Crow Laboratory of Physics, Washington University, St. Louis, Missouri (Received September 8, 1958).

A new variational approach is developed for studying the properties of systems of particles interacting through singular short-range repulsions that give rise to strong two-particle correlations. The correlated trial function $\Psi_\gamma = e^S \Phi_\gamma$ (state γ) results, with proper choice of S, in a simple form for the energy expectation value $\langle H \rangle$– as well as other matrix elements of interest–which is devoid of all reference to the strong repulsions except through e^{2S} factors and hence particularly suited to calculation. In many cases an independent-particle type Φ_γ seems appropriate. The cluster evaluation of this form for $\langle H \rangle$ is discussed, both in the few-particle and many-particle cases. Using the techniques of Iwamoto and Yamada, simplified convergent cluster expansions for the energy expectation value are derived for many-fermion and many-boson systems. A program for application of this method to nuclear problems is being initiated.

GENERAL RELATION BETWEEN FLUXES FROM COLLIMATED POINT AND PLANE SOURCES OF RADIATION. Martin J. Berger and Lewis V. Spencer, National Bureau of Standards, Washington, D. C. (Received June 23, 1958)

The radiation density (flux integrated over all directions) from a collimated point source is shown to be representable as a Fourier-Hankel transform of the density from a collimated plane oblique source. The spatial moments of a point-source density are similarly expressible as linear combinations of a finite number of moments of plane-source densities corresponding to different source obliquities. These results are valid for any type of radiation, provided that the medium is unbounded, homogeneous, and isotropic, and provided that a linear transport equation applies. The plane-geometry representation of densities and density-moments is equivalent to a "separation of variables." It allows one to solve a sequence of problems in one space variable (distance from a source plane) for different values of the source obliquity, and to use the information thus obtained for constructing the solution of a problem involving two space variables (e.g., the longitudinal and radial distances from a point source).

SOUND VELOCITY AND ADIABATIC COMPRESSIBILITY OF LIQUID HELIUM-THREE. Henry L. Laquer, Stephen G. Sydoriak, and Thomas R. Roberts, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received August 22, 1958).

The velocity of first sound at 5 Mc/sec was measured in liquid He^3 between 0.34 and 3.14°K. The results are fitted within 0.2% by the equation $u = 183.9 - 5.98T^2 - 0.130T^3 - 0.00176T^8$ (m/sec). Initial pressure coefficients of the sound velocities and a number of derived thermodynamic quantities are also reported.

TRAJECTORY-WISE ANALYSIS OF CYLINDRICAL AND PLANE PLASMAS IN A MAGNETIC FIELD AND WITHOUT COLLISIONS. Lewi Tonks, Radiation Laboratory, University of California, Livermore, California (Received September 19, 1958).

The anatomy of the transition region between vacuum and a fully developed magnetically immobilized ionized-gas plasma has been examined by following particle trajectories in detail. The mathematical formulation has required machine computation for its full interpretation. This approach recognizes the structure imparted to the plasma by the radius of gyration and thus serves as a critique of the magnetohydrodynamic method. It furnishes the microscopic verification of that macroscopic approach and supplements it by showing that the sharpness of transitions in a plasma are limited, in effect, by the gyration radius in the stronger, not the weaker, magnetic field. Especially, it brings out the greater importance of the tensor character of the plasma

pressure in the cylindrical case where the combined kinetic and magnetic energy density in uniform interior regions of the plasma is not equal to the magnetic energy density in the vacuum. The analysis also exhibits the intense mass motion at the surface of a strong plasma constituting a paramagnetic electric current and probably having dynamical effects. The numerical work has been correlated where possible with direct theoretical results.

THERMODYNAMICS OF INHOMOGENEOUS SYSTEMS. Edward W. Hart, General Electric Research Laboratory, Schenectady, New York (Received August 11, 1958).

A self-consistent thermodynamic formalism is developed for the treatment of the equilibrium of systems, some of whose parameters vary continuously from place to place. The method is specially designed for the description of transition interfaces separating two phases. The energy per unit volume is assumed to depend explicitly on the space derivatives of the molecule densities. Equilibrium conditions are obtained for the appropriate internal variables of the system, and all externally measurable intensive variables are uniquely defined by a variational procedure.

NONLINEAR ELECTRON OSCILLATIONS IN A COLD PLASMA. John M. Dawson, Project Matterhorn, Princeton University, Princeton, New Jersey (Received September 19, 1958).

Investigations of nonlinear electron oscillations in a cold plasma where the thermal motions may be neglected indicate that except for the simplest one-dimensional situation such oscillations will destroy themselves through the development of multistream flow. It is found possible to give an exact analysis of oscillations with plane, cylindrical, and spherical symmetry. Plane oscillations in a uniform plasma are found to be stable below a critical amplitude. For larger amplitudes it is found that multistream flow or fine-scale mixing sets in on the first oscillation. Oscillations with spherical or cylindrical symmetry develop multistream flow almost always, independent of the amplitude. The time required for mixing to start is inversely proportional to the square of the amplitude. Plane oscillations in a nonuniform plasma are also found to exhibit this type of behavior. Some considerations are also given to more general oscillations and a calculation is presented which indicates that multistream flow will usually set in.

METALLIC TRANSITION IN LITHIUM HYDRIDE. Robert E. Behringer, Department of Physics, University of California, Berkeley, California, and International Business Machines Research Laboratory, Poughkeepsie, New York (Received September 8, 1958).

The dependence on lattice parameter, or pressure, of the energy gap between the last filled and first empty energy band in LiH is investigated. The crystalline potential is approximated by a Wigner-Seitz atomic-spheres potential, corrected to account for the effects of overlapping spheres, and the wave functions are expanded in symmetrized plane waves. In addition the effect of varying the relative size of the lithium and hydrogen ion spheres is considered. The energy levels at the top of the filled band and the bottom of the empty band are determined at several values of the lattice parameter, using 6, 7, 8, 9, and 10 symmetrized plane waves. The problem of convergence is examined.

The model predicts that transition to the metallic state should occur at a pressure of about 35 megabars. The results are discussed in light of recent experimental investigations.

$M_{2,3}$ EMISSION BAND OF COPPER. D. E. Bedo and D. H. Tomboulian, Cornell University, Ithaca, New York (Received September 15, 1958).

The copper $M_{2,3}$ band was investigated in connection with a sequential study of the $M_{2,3}$ emission and absorption spectra of the $3d$ transition metals. The characteristic band is superimposed on a continuous spectrum emitted by the target and the paper describes the procedure for removing the effect of this background. Other experimental considerations dealing with such effects as the instrumental window, self-absorption, the presence of contaminants, and the nature of the target layer are included. Where possible, an assessment is made of the distortion produced by such factors in the shape of the band. The photometrically determined $M_{2,3}$ intensity contour is compared with recent work based on detection schemes utilizing photon counters. On the basis of certain assumptions, the composite $M_{2,3}$ distribution is decomposed

into individual M_2 and M_3 curves, whose behavior at the high-energy end of the spectrum is in good agreement with previously available data on the individual absorption edges. Features of the $I(E)/\nu^3$ distribution belonging to the M_3 band are as follows: the band has a maximum at 72.0 ev (172 A) and excluding satellite structure and tailing effects has a width of 7.8 ev. At the high-energy end, the intensity rises gradually and no sharp structures are noticeable. Except for the satellite structure, no inconsistencies arise when the results on the M_3 band (valence→3p) are compared with those on the L_3 band (valence→2p).

TEMPERATURE DEPENDENCE OF THE BREAKDOWN FIELD OF BARIUM TITANATE. P. H. Fang and W. S. Brower, National Bureau of Standards, Washington, D. C. (Received July 28, 1958).

The dc breakdown field has been measured for ceramic $BaTiO_3$ as a function of the ambient temperatures. The breakdown field shows, in general, a temperature dependence of the thermal breakdown type where the breakdown field decreases as the temperature is increased. However, in addition to this general behavior, the breakdown field dips down strongly and shows minima at the critical temperatures of the phase transformations. This seems to indicate a process of polarization breakdown, and is most pronounced near the orthorhombic-tetragonal transition temperatures.

EFFECTIVE IONIC CHARGE IN ALKALI HALIDES. Jacque E. Hanlon and A. W. Lawson, Department of Physics and Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received September 22, 1958).

An expression is derived for the effective charge in the Born-Szigeti equation for the dielectric constant of ionic crystals with the NaCl structure. The derivation is based on a coupled oscillator model in which the lattice vibrations are coupled to the electronic motions within the ions. The deviation of the ionic charge from its nominal value is found to be proportional to the difference in electronic polarizability between the positive and negative ions. Agreement between theory and experiment is satisfactory. Secondary effects on the index of refraction and the reststrahl frequency are calculated. The model is in semiquantitative agreement with the observed dipole moments of alkali halide molecules.

CONDUCTIVITY OF GRAIN BOUNDARIES IN GROWN GERMANIUM BICRYSTALS. B. Reed, O. A. Weinreich, and H. F. Mataré, Research Laboratories, Sylvania Electric Products, Incorporated, Bayside, New York (Received September 22, 1958).

The conduction of current in the grain boundary of a grown germanium bicrystal has been studied as a function of doping. For all samples, the behavior shows only a small temperature dependence from 2 to 300°K. The conduction in the grain boundary is ohmic if no secondary effects are introduced by conduction in the bulk material. Samples with no doping and with n-type, p-type, or copper doping are all characterized by having a resistivity of about 3000 to 11 000 ohms per square. The lack of a strong dependence on doping indicates that the grain boundary behavior is not due to the segregation of impurities at the boundary.

HALL EFFECT FOR ELECTRONS IN SILVER CHLORIDE. Koichi Kobayashi and Frederick C. Brown,* Department of Physics, University of Illinois, Urbana, Illinois (Received September 22, 1958).

The Hall mobility of photoelectrons has been measured down to 5°K in single crystals of silver chloride. The experiment was carried out with a pulse technique which overcomes the difficulties of space charge and high impedance common to such measurements on insulating crystals. Results show that the electron Hall mobility rises to high values, 6000 cm^2/volt-sec, at very low temperature. On the other hand, drift mobility is strongly influenced by multiple trapping and is not characteristic of microscopic mobility in these crystals at low temperature. Comparison with mobility theory indicates that electrons are scattered by both the optical and acoustical modes of lattice vibration depending upon the range of temperature. Evidence for a structure-dependent scattering such as would be produced by impurities is found below 20°K. Magnetoresistance effects begin to become important at high values of mobility and magnetic field in qualitative agreement with the theory of

electronic conductivity for a cubic crystal with simple energy bands and non-degenerate statistics.

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X-RAY *K*-ABSORPTION SPECTRA OF SOLID ARGON AND KRYPTON. Jack A. Soules* and C. H. Shaw, Department of Physics, The Ohio State University, Columbus, Ohio (Received September 24, 1958).

The x-ray *K* absorption edge structures of argon (at 3.86 A) and of krypton (at 0.86 A) have been measured in the gaseous and solid states. The spectra of the gases confirm previous work. The spectra of the solids exhibit pronounced structure extending far to the short-wavelength side of the absorption edge. The broad absorption maxima observed can be correlated with the valence bands in the crystal, but the extremely sharp initial rise requires an unusual density of states in the bands, perhaps offering evidence for the existence of excitation states.

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LOW-TEMPERATURE ELECTRICAL AND MAGNETIC BEHAVIOR OF DILUTE ALLOYS: Mn IN Cu AND Co IN Cu. I. S. Jacobs and R. W. Schmitt, General Electric Research Laboratory, Schenectady, New York (Received September 22, 1958).

Measurements are reported of the remanent magnetization in a 1.8-atomic percent Mn in Cu alloy at 4.2°K and of the low-temperature resistance, magnetoresistance, and magnetization of solid solution alloys of 0.5, 1.0, and 2.0 atomic percent Co in Cu. The isothermal remanence of the Cu(Mn) alloy saturates at about 3×10^{-2} (μ_B/Mn atom) after application of a field of about 14 kilo-oersteds, and does not increase after application of pulsed fields up to 140 kilo-oersteds. It is equal to the saturation thermoremanence obtained after cooling in fields $\geq$1.5 kilo-oersteds. Both remanences reverse in relatively low fields of about 2 kilo-oersteds. The magnetoresistance and magnetization of the Cu(Co) alloys obey the relation, $\Delta\rho = -b\sigma^2$, found previously for Cu(Mn) alloys, with b temperature-independent in contrast to that for Cu(Mn). The low-temperature resistivity of the Cu(Co) alloys increases with decreasing temperature and no maximum occurs down to 1.6°K. The magnetization of the Cu(Co) alloys shows neither remanence nor hysteresis but is nonlinear with field and with concentration. Comparison of results on these systems with existing theories delineates the areas of agreement and of disagreement.

STORED-ENERGY RELEASE IN COPPER FOLLOWING ELECTRON IRRADIATION BELOW 20°K. C. J. Meechan and A. Sosin, Atomics International, A Division of North American Aviation, Incorporated, Canoga Park, California (Received September 25, 1958).

The stored-energy release in copper has been measured in the temperature range 20°K - 60°K following irradiation with 1.2-Mev electrons. A differential temperature measurement was made between an irradiated specimen and an unirradiated standard. The specimens were immersed in liquid helium during irradiation; subsequent heating of the specimen was carried out in vacuum. A value of the total energy release of 2.5×10^{-2} cal/g was observed for an integrated flux of 9×10^{17} e/cm^2. The stored energy-resistivity ratio obtained is (5.4 ± 0.8) cal/g per microhm-cm. The energy associated with a Frenkel pair is calculated to be (5.4 ± 0.8) ev for a value of 3.6 microhm-cm per atomic percent Frenkel defects.

EFFECT OF PRESSURE ON ANELASTIC RELAXATION IN SILVER-ZINC. G. W. Tichelaar and D. Lazarus, Department of Physics, University of Illinois, Urbana, Illinois (Received September 22, 1958).

Using a novel experimental apparatus, studies have been made of the effect of hydrostatic pressure up to 9000 kg/cm^2 on the rate of stress relaxation at constant small strain in an Ag-30 atomic percent Zn alloy, where the relaxation process is presumably diffusion limited. For temperatures between 110°C and 150°C the relaxation time is found to increase exponentially with pressure. The zero-pressure data are in good agreement with previous work by Nowick. At 9000 kg/cm^2 the relaxation time is about a factor of four greater than at 1 kg/cm^2, for all temperatures. The pressure dependence can be

interpreted in terms of an "activation volume" of 5.36 ± 0.07 cm^3/mole, which is about half the atomic volume of the material. This may mean that the volume of formation of a vacancy is at most about half of the molar volume, which number is in accordance with a recent calculation by Tewordt.

OPTICAL SPECTRUM OF Co^{2+} IN THE CUBIC CRYSTALLINE FIELD OF CaF_2. R. Stahl-Brada and W. Low, Department of Physics, The Hebrew University, Jerusalem, Israel (Received September 16, 1958).

The optical spectrum of Co^{2+} in CaF_2 is analyzed. Absorption bands are found at 3 μ, 1.5 μ, and 550 mμ. These bands show fine structure. In addition there are weak and narrower bands at 19 220, 19 580, 20 450, 21 750, 22 000, and 25 750 cm^{-1}, and intense lines at 44 250 and 48 200 cm^{-1}. The spectrum is interpreted to arise from transitions from the orbital singlet to various Stark levels split by spin-orbit interaction, and yields values of $Dq = 340 \pm 10$ cm^{-1}, $E(P) = 14\,000 \pm 100$ cm^{-1}, and $E(G) = 16\,250 \pm 100$ cm^{-1}. The spectrum of Co^{2+} in the eight-coordinated cubic symmetry is compared with Co^{2+} in octahedral and tetrahedral symmetry as well as with Cr^{3+} in octahedral symmetry.

NEUTRON DIFFRACTION INVESTIGATIONS OF THE MAGNETIC ORDERING IN $FeBr_2$, $CoBr_2$, $FeCl_2$, AND $CoCl_2$. M. K. Wilkinson, J. W. Cable, E. O. Wollan, and W. C. Koehler, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received September 8, 1958).

Neutron diffraction experiments have been performed on anhydrous $FeBr_2$, $CoBr_2$, $FeCl_2$, and $CoCl_2$ at temperatures from 295°K to 4.2°K to investigate the existence of magnetic ordering in these hexagonal layer-type structures. All four compounds have an antiferromagnetic transition at low temperatures to structures in which the atomic magnetic moments within a metal layer form ferromagnetic sheets and the moments in adjacent layers are antiparallel. In the iron compounds the moments are oriented parallel to the hexagonal c axis and in the cobalt compounds the moment orientation is perpendicular to that axis. Values of the atomic magnetic moments are close to those expected for the divalent metallic ions if the orbital contribution is quenched. Small-angle scattering experiments on $FeCl_2$ and $CoCl_2$ have shown that the ferromagnetic coupling between moments within a layer is much stronger than the antiferromagnetic coupling between atoms in adjacent layers, and single-crystal investigations on these two compounds have determined the method by which large net magnetization values are obtained at temperatures below T_N in moderate magnetic fields.

METHOD OF TREATING ZEEMAN SPLITTINGS OF PARAMAGNETIC IONS IN CRYSTALLINE FIELDS. G. F. Koster, Massachusetts Institute of Technology, Cambridge, Massachusetts, and H. Statz, Raytheon Manufacturing Company, Waltham, Massachusetts (Received September 17, 1958).

In this paper, we present an alternative method to the "spin Hamiltonian" for treating the behavior of a paramagnetic ion under the combined influence of the host crystal and an applied magnetic field. This method has the advantage of being applicable to all strengths of couplings between the paramagnetic ion and the surrounding crystal. It is applicable to free-ion S states in the same way that it applies to all other states. It gives all of the constants one needs to describe the energy level structure as a function of field as restricted by symmetry and the properties of the levels considered. The method is more general and is expected to be also more accurate than the conventional spin Hamiltonian formalism. It is illustrated by the application to the problem of the $(d)^5$ ^6S level in a cubic crystalline environment.

7.65-Mev STATE OF C^{12}. S. F. Eccles* and D. Bodansky, University of Washington, Seattle, Washington (Received September 22, 1958).

The angular distribution for the inelastic scattering of 42-Mev alpha-particles by C^{12}, with excitation of the 7.65-Mev state, has been measured. The experimental distribution shows maxima and minima consistent with a 0^+ or 2^+ spin-parity assignment for the 7.65-Mev state, on an Austern, Butler, McManus type of direct-interaction theory. A search was made for coincidences between the inelastically scattered alpha particles and recoil carbon nuclei in an attempt to determine the probability that the 7.65-Mev state decays by transitions to the

ground state of C^{12}. It was found that there is less than one chance in ten that this probability exceeds 0.1%. This low probability is not inconsistent with current theories of helium burning in stars and provides additional support for the usual 0^+ assignment for the 7.65-Mev state.

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DISINTEGRATION OF I^{124} AND I^{123}. Allan C. G. Mitchell, Jose O. Juliano, Charles B. Creager, and C. W. Kocher, Physics Department, Indiana University, Bloomington, Indiana (Received September 8, 1958).

The disintegration of I^{124} (4.2 days) has been studied with the help of magnetic spectrometers and scintillation spectrometers. The disintegration occurs 71% by electron capture and 29% by positron emission. Three positron groups were found having end-point energies of 2130 (46.0%), 1531 (46.4%) and 786 (7.5%) kev. The most energetic positron group has a shape characteristic of $\Delta I = \pm 2$, yes. Positron-gamma coincidence experiments show that this group goes to the ground state. Gamma rays of energies 2700, 2300, 2100, 1700, 1520, 1350, 723, and 603 kev together with annihilation radiation and Te K x-rays have been found and the relative intensities measured. A disintegration scheme, consistent with the levels of Te^{124} as determined from the decay of Sb^{124}, has been established. No beta rays were found showing that a transition to Xe^{124} is highly improbable. The former work on I^{123} (13.5 hours) has been substantiated and, in addition, it seems highly unlikely that any positrons are emitted from I^{123}.

FISSION NEUTRON SPECTRUM AND NUCLEAR TEMPERATURES. James Terrell, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received August 7, 1958).

It is shown that Weisskopf's nuclear evaporation theory, when allowance is made for the expected distribution of nuclear temperatures of fission fragments, predicts an essentially Maxwellian distribution of fission neutron energies in the laboratory system. This is found to be in excellent agreement with all available data. On the assumption that neutron emission is symmetrical about 90° in the center-of-mass system, the average energy $\overline{E}$ of the fission spectrum should be $\overline{E} = \overline{E}_f + 2\overline{T}$, in which $\overline{E}_f$ and $\overline{T}$ are the average values of the fission fragment energy per nucleon and the nuclear temperature. Experimentally, $\overline{E}_f \cong 0.78$ Mev for all cases reported, giving fission fragment nuclear temperatures of 0.6 to 0.7 Mev for measured fission neutron spectra. This gives $a = 12 \pm 2$ Mev^{-1} for the equation $E_e = aT^2$=excitation energy. The same concepts lead to the prediction $\overline{T} \cong \frac{2}{3}[(\overline{\nu}+1)E_0/2a]^{1/2}$, or $\overline{E} \cong 0.78$ Mev $+ 0.621\,(\overline{\nu}+1)^{1/2}$ for $U^{235} + n$; E_0 is the excitation energy change per emitted neutron, about 6.7 Mev, and $\overline{\nu}$ is the average number of neutrons emitted per fission. This equation, which is approximately valid for all present experimental data, leads to the prediction that $d\overline{E}/dE_x \cong 0.025$ for U^{235}(E_x is the excitation energy of the fissioning nuclide). The center-of-mass energy spectrum of fission neutrons has also been calculated, as well as effects of anisotropy of emission on the laboratory fission neutron spectrum.

DECAY OF THE RADIOISOTOPES Ge^{68} AND Ga^{68}. Daniel J. Horen, Stanford University, Stanford, California (Received August 18, 1958).

The decays of Ge^{68} (275-day) and Ga^{68} (68-minute) have been reinvestigated with single and coincidence scintillation spectrometers. Comparison of the gamma-ray singles spectra of the two radioisotopes indicates that Ge^{68} decays only to the ground state of Ga^{68} by electron capture. In addition to the known annihilation quanta and a gamma ray of 1.07 Mev in the decay of Ga^{68}, gamma rays of 0.81±0.02, 1.24±0.02, and 1.88±0.02 Mev are observed. A level scheme of Zn^{68} with levels at 1.07, 1.88, and 2.31 Mev is proposed and is compared with level schemes of neighboring even-even nuclei.

NUCLEAR REACTIONS INDUCED BY THE NITROGEN BOMBARDMENT OF SULFUR. D. E. Fisher * and A. Zucker, Oak Ridge National Laboratory, Oak Ridge, Tennessee, and A. Gropp, University of Florida, Gainesville, Florida (Received September 29, 1958).

Thick targets of ZnS, containing natural sulfur, were bombarded with 28-Mev nitrogen ions in the ORNL 63-inch cyclotron. The compound nucleus resulting from bombardment of sulfur-32 with 28-Mev nitrogen ions is V^{46}, with an excita-

tion energy of 33.3 Mev. The following nuclear reactions were studied: (1) $S^{32}(N^{14}, p)Ti^{45}$; (2) $S^{32}(N^{14}, 2p)Sc^{44}$; (3) $S^{32}(N^{14}, 2p)Sc^{44m}$; (4) $S^{32}(N^{14}, 2pn)Sc^{43}$; (5) $S^{32}(N^{14}, 2\alpha)K^{38m}$; and (6) $S^{32}(N^{14}, N^{13})S^{33}$. The yields as a function of incident energy for these reactions were differentiated to obtain excitation functions. The cross sections for (1), (2 and 3), and (5) are compared with calculations based on the statistical theory of compound nucleus decay. Two values of the level density parameter, a, were employed, first considering and then ignoring de-excitation of a nucleus by gamma-ray emission. The value $a = A/10.5$ fits the p and $2p$ emission cross sections when gamma-ray emission is considered. The excitation function for reaction (6) agrees well with the systematics of previously studied nitrogen-induced transfer reactions.

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NITROGEN-INDUCED NUCLEAR REACTIONS IN POTASSIUM. J. J. Pinajian and M. L. Halbert, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received September 22, 1958).

Excitation functions have been measured for the production of Fe^{53}, Fe^{52}, Mn^{52}, Mn^{52m}, Mn^{51}, Cr^{49}, V^{48}, V^{47}, and Ti^{45} from the nitrogen bombardment of potassium in its normal isotopic mixture. The method used was activation of thick targets of KBr followed by chemical separations and absolute beta-counting. The cross sections at 27.5 Mev range from 0.14 mb for Fe^{52} to 8.3 mb for Mn^{51}. The relation of the Mn^{52}:Cr^{49}:Fe^{52} ratio to the odd-even effect in the nuclear level density is discussed. The total cross section for formation of the compound nucleus was estimated at several energies. The fraction of this total accounted for by one-particle emissions from the compound nucleus is surprisingly large.

INELASTIC PROTON SCATTERING IN MEDIUM-WEIGHT ELEMENTS. Bernard L. Cohen* and Allen G. Rubin, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received September 29, 1958).

Measurements were made of the energy distribution of emitted protons from nuclear reactions induced by protons of various energies between 11 and 23 Mev in elements of atomic number 22 to 30. At intermediate energies, the spectrum shows two maxima; there is strong evidence that the low-energy maximum is due either to "second protons," or to simultaneous emission of two particles. Measurements of nuclear temperatures at 18 Mev are strongly distorted by these effects. At lower bombarding energies, the low-energy contribution may be subtracted off, and distortion of the high-energy part of the spectra by direct ejection is not large, so that the statistical theory of nuclear reactions may be studied. Values obtained for the level density parameter a are independent of the energy of the emitted particle, independent of bombarding energy, slowly varying with atomic weight, and consistent with values obtained from neutron-induced reactions. But when these values are used to calculate cross sections for (n, p) and (p, p') reactions, it is found that protons are emitted with excessive probability in the latter. Some possible explanations for this are discussed.

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SPACINGS OF NUCLEAR ENERGY LEVELS. Lawrence Dresner, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received September 29, 1958).

A simple statistical model is suggested in terms of which the general features of the level spacing distribution can be understood, and which involves no special assumptions, other than that of Porter and Thomas.

BETA DECAY OF Be^{11}. D. H. Wilkinson* and D. E. Alburger, Brookhaven National Laboratory, Upton, New York (Received September 30, 1958).

Be^{11} has been made in the reaction $B^{11}(n, p)Be^{11}$ and its radiations have been studied with plastic and NaI(Tl) scintillation spectrometers both singly and in coincidence. The half-life as determined from measurements on both beta rays and gamma rays is 13.57 ± 0.15 sec. Decay takes place by beta-ray emission to the ground state of B^{11} (61%, $\log ft = 6.77$, $\beta_{max} = 11.48$ Mev), to the 2.14-Mev first excited state (29%, $\log ft = 6.63$), to the 6.76-6.81-Mev doublet (6.5%, $\log ft = 5.93$) and to the 7.99-Mev level (4.1%, $\log ft = 5.53$). Limits are set on the decay to several other levels, particularly those at 4.46

and 5.03 Mev ($\leq 0.2\%$). Gamma rays of 2.121, 4.64, 5.86, 6.76, and 7.97 Mev are observed both singly and in coincidence with beta-ray branches. An energy separation $Be^{11} - B^{11} = 11.48 \pm 0.15$ Mev is derived from the various beta- and gamma-ray energy measurements. It is concluded that the assignment $J = \frac{1}{2}^-$ for Be^{11}, as expected from shell model, is possible but cannot be established firmly on the basis of the present evidence.

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β-γ CORRELATIONS IN THE FIRST-FORBIDDEN TRANSITION. Tsuneyuki Kotani* and Marc Ross, Department of Physics, Indiana University, Bloomington, Indiana (Received September 25, 1958).

A discussion is given of the theory of β-γ correlations in the unique and nonunique first-forbidden transition. In particular, numerical results are presented for the Coulomb corrections to the β-γ directional correlation and transverse β polarization-γ correlation. The energy dependence of the contribution associated with violation of time-reversal invariance is given.

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HYDRODYNAMIC THEORY OF SPONTANEOUS FISSION. W. D. Foland* and R. D. Present, University of Tennessee, Knoxville, Tennessee (Received September 8, 1958).

The penetration factor for spontaneous fission has been calculated from the liquid-drop model. The transformation of the Gamow integral over the nucleon coordinates into an integral over the deformation parameters a_n has been carried out hydrodynamically, assuming irrotational motion. The transformation requires evaluation of the kinetic energy in terms of a_n and $\dot{a}_n$. Series expansions are used for the kinetic energy and for the potential energy of deformation. We have neglected all parameters but a_2 and carried the hydrodynamic calculations through terms in a_2^4. While the potential barrier is subject to several uncertainties, it has nevertheless been possible to estimate the spontaneous fission hindrance factor for the highest Z elements. We find for $Z = 100$ and $Z^2/A \approx 39$ that a 1-Mev increase in barrier height should correspond to a $10^{3.7}$-fold increase in the half-life. This result agrees closely with the empirical hindrance factor formula deduced by Swiatecki from a correlation of fluctuations in half-lives with deviations of ground-state masses from the semiempirical mass formula. We have included some details of both the hydrodynamic and the electrostatic calculations.

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INELASTIC SCATTERING OF PROTONS BY VARIOUS NUCLEI. T. Tamura* and D. C. Choudhury, Department of Physics, University of California, Los Angeles, California (Received July 7, 1958).

Experimental data on the inelastic scattering of 23-Mev protons reported by Cohen, Mosko, and Rubin are analyzed by using the theory of direct interaction together with the plane-wave approximation. The most detailed analysis is done for lead isotopes and, at least at 90°, it is found that fairly good agreement with experiment is obtained for the relative magnitudes of the excitation of different excited states. The same idea is also applied to other cases and in particular several arguments are given which will support the view of Lane and Pendlebury for associating the anomalous peaks observed at ≈ 2.5 Mev for nuclei with $Z = 30$-53 with the octupole surface vibrations. Finally discussions are given of the validity of the approximations used in the present analysis and also of the circumstances which still need more refined calculations.

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DISINTEGRATION OF IRON-52 AND IRON-53. Jose O. Juliano, C. W. Kocher, T. D. Nainan, and Allan C. G. Mitchell, Physics Department, Indiana University, Bloomington, Indiana (Received September 15, 1958).

The disintegration of Fe^{52} (8.2 hours) and Fe^{53} (8.9 min) has been investigated with the help of scintillation and coincidence counting equipment. Fe^{52} decays 56.5% by positron emission and 43.5% by electron capture. The end-point energy of the positron group is 0.804 ± 0.01 Mev. This is followed by a gamma ray of energy 165 kev leading to Mn^{52m} (21 min). The chain $Fe^{52} \rightarrow Mn^{52} \rightarrow Cr^{52}$ has been studied. In addition to the well-

known states of Mn^{52} – the ground state, with character 6+ and half-life of 5.7 days, and the first excited state (Mn^{52m}), with character 2+, half-life of 21 min, and energy of 390 kev – these experiments show a third excited state at 555 kev having a configuration 0+ and a half-life of $(1.2 \pm 0.2) \times 10^{-8}$ sec. The disintegration of Fe^{53} is accompanied by the emission of a gamma ray of energy 380 kev and positron groups of end-point energies 2.84 ± 0.10, 2.38 ± 0.10 Mev, and an indication of a third group at 1.57 ± 0.15 Mev.

INHIBITION OF MAGNETIC DIPOLE RADIATION AND THE IDENTIFICATION OF T=1 STATES IN LIGHT, SELF-CONJUGATE NUCLEI. E. K. Warburton, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received September 22, 1958).

The effects of Coulomb impurities on the inhibition rule of Morpurgo for ΔT=0, $M1$ transitions in light ($A \leq 20$), self-conjugate nuclei are discussed, and a comparison is made between Morpurgo's rule and the experimentally known $M1$ transition strengths. An upper limit to the average inhibition of ~10 is indicated by the experimental evidence. There are no known $\Delta T = 0$, $M1$ transitions in light, self-conjugate nuclei with strengths greater than 0.1 Weisskopf units. The work of Morpurgo is shown to lead to spin-dependent upper limits of the order of $0.5 \times 10^{-2} A(A+2)$ Weisskopf units for the matrix elements of ΔT=0, $M1$ transitions in self-conjugate nuclei. These limits are invoked to assign isotopic spin to several levels in B^{10}, N^{14}, and F^{18}.

BREAKUP OF DEUTERONS BY PROTONS. L. Cranberg and R. K. Smith, University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (Received September 22, 1958).

Results are given for the neutron spectra at zero degrees from the reaction $p + d \rightarrow 2p + n$ for a range of incident proton energies from $E_p = 4.0$ to $E_p = 7.0$ Mev. Also given are the spectra as a function of angle for $E_p = 6.5$ Mev. The observed spectral shapes show systematic discrepancies from those given by the theory of Frank and Gammel; the discrepancies increase with increasing proton energy although the reaction yield is accurately given by the theory.

CAPTURE AND DECAY OF μ^- MESONS IN Fe. W. A. Barrett,* F. E. Holmstrom, and J. W. Keuffel, University of Utah, Salt Lake City, Utah (Received September 2, 1958).

The mean life of μ^- mesons in Fe has been measured using an improved cosmic-ray apparatus. A positive identification of the stopped muon was made using Čerenkov velocity selectors in the incident telescope. The 2.2-μsec background from positive muons was reduced a factor of 3 with a 3-layer sandwich of Fe and thin plastic scintillators, so arranged that electrons emitted in the target were mostly detected as such by the scintillators. The mean life is 196 ± 8 mμsec. By comparing this result with the electron-counting results of Lederman and Weinrich, the ratio of the decay rate of μ^- bound in Fe to the free μ^+ decay rate is found to be 1.15 ± 0.06.

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DETERMINATION OF THE PION-NUCLEON COUPLING CONSTANT FROM PHOTOPRODUCTION ANGULAR DISTRIBUTION. John G. Taylor,* University of California Radiation Laboratory, Berkeley, California, Michael J. Moravcsik, University of California Radiation Laboratory, Livermore, California, and Jack L. Uretsky, University of California Radiation Laboratory, Berkeley, California (Received September 22, 1958).

The pion-nucleon coupling constant is determined from the pion photoproduction angular distribution. The method is based upon a certain conjecture concerning the analyticity of the photoproduction amplitude, and does not depend on the validity of any specific theory of photoproduction. It consists of an extrapolation of the angular distribution at any given fixed energy to $\cos\theta = V_\pi^{-1}$, where V_π is the pion velocity divided by the velocity of light. The amplitude at this nonphysical angle has a pole, and the pion-nucleon coupling constant is simply related to the residue of the amplitude. The quartic representation of photoproduction angular distributions is used as the functional form of the extrapolating curve. The most important feature of the new method is in the fact that it measures, at least in principle, the pion-nucleon coupling constant at any given fixed energy, while pre-

vious determinations in general measure the coupling constant in the low-energy limit, or require assumptions concerning the behavior of the cross section at all energies.

In addition, the new method does not depend on the assumption of charge independence, and in fact measures explicitly the interaction of positive pions with nucleons. The scheme cannot be used for photoproduction of neutral pions. The method is applied to available data at 230, 260, 265, and 290 Mev photon energy in the laboratory system, and an over-all value of $f^2 = 0.064 \pm 0.041$ is obtained. In view of the large error, a detailed discussion is given of possible improvements in experiments which could give a more accurate value. Also discussed is the sensitivity of the value of the coupling constant to various features of the experiment, such as the energy of the photons, the relative importance of the various angles, the relative importance of the relative and absolute normalizations, and the statistical errors on the individual pieces of data. Finally, numerical illustrations are given of the accuracy obtainable for certain given conditions on the factors listed above.

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EVIDENCE AGAINST THE MASS-500 PARTICLE. E. Bierman, R. Lea,* J. Orear,† and S. Rosendorff, Department of Physics, Columbia University, New York, New York (Received September 29, 1958).

The existence of a mass-500 particle was investigated by exposing a nuclear emulsion to cosmic rays at an altitude of 10 600 feet and geomagnetic latitude 39°. Particles of average range 60 g/cm² of copper were stopped in the emulsion and grain counted at a residual range of 1 cm. The available track length was such that the existence of a mass-500 particle would have been established on the basis of a single occurrence. About 1100 muons were found with no particles in the region of 500 electron masses being observed. This is not consistent with the abundance reported by Alikhanian et al., since the probability of finding none, assuming their reported abundance, is 0.4%.

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POINT SOURCE OF COSMIC RAYS IN ORION. Y. Sekido, S. Yoshida and Y. Kamiya, Physical Institute, Nagoya University, Nagoya, Japan (Received March 31, 1958; revised manuscript received October 22, 1958).

By using Geiger-Müller counter telescopes with alt-azimuth mountings, a point source of cosmic rays was found. The telescopes were kept nearly horizontal so as to observe only high-energy cosmic rays which could pass through the thick layer of the atmosphere. After scanning the celestial sphere, a small part of it was observed again and again during the period from April 1954 to December 1956, and thus the existence of a point source was established. Declination of the point source was about 0.5°N and right ascension was about 5 hr 15 min. The average momentum of the primary particles effective for this observation would be about 280 Gev/c if they are protons. In this momentum range, the time average of the intensity was about 10% of the background cosmic rays, while there were some periodic variations during the period of observation.

RADIATIVE CAPTURE OF K^- MESONS FROM ORBITAL P STATES OF K-MESONIC ATOMS. Graham Frye, Radiation Laboratory, University of California, Berkeley, California (Received September 22, 1958).

The photon spectrum accompanying the radiative capture of K^- mesons from orbital P states of K-mesonic hydrogen atoms is discussed.

IONIZATION LOSS BY μ MESONS IN HELIUM. Robert E. Lanou, Jr.,* and Henry L. Kraybill, Yale University, New Haven, Connecticut (Received September 29, 1958).

The ionization loss by cosmic-ray μ mesons in helium gas has been measured as a function of momentum. The ionization loss was determined with proportional counters and the momenta were measured by a magnetic spectrometer which resolved particles in the momentum region from 3.3 Bev/c to 140 Bev/c. It was found that helium gas at 2.7 atmospheres pressure exhibits a density-effect saturation of the most probable ionization loss and that this saturation is complete at a $p/\mu c$ value of about 200. Under the conditions of normalization used in this experiment, the value of the ionization loss

at which the Fermi plateau occurs is 1.28 ± 0.04 times the value at the minimum. This is in agreement with calculations based on the Sternheimer theory for the particular counter filling used in this experiment.

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ELASTIC SCATTERING OF 150-Mev NEGATIVE PIONS BY NUCLEI. Tadao A. Fujii, The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois (Received September 2, 1958).

Elastic scattering of 150-Mev negative pions by complex nuclei was experimentally studied by the use of energy-sensitive Cerenkov detectors with a pulse-height analyzer. Elastic scattering could be distinguished from inelastic processes within the 10-Mev resolution width of these detectors. The differential cross sections of carbon, aluminum, copper, and lead were measured at angles between 18.5° and 43.6°. In addition, measurements on carbon and lead were extended to large angles between 45° and 139°.

It was found that elastic scattering was confined predominantly to the forward angles less than 60°. Calculations based on the optical model with a square well were carried out to obtain the values of parameters which provided the best fit to the data. They are as follows: -30 Mev $\geq V_R \geq -40$ Mev, -65 Mev $\geq V_I \geq -75$ Mev, and $1.3\times10^{-13} \times A^{1/3}$ cm $\leq R \leq 1.4\times10^{-13} A^{1/3}$ cm, where V_R and V_I are real and imaginary parts of the potential, R is the nuclear radius, and A is the nuclear weight. The corresponding values of the reaction mean free path were of the order of the pion Compton wave length. These values are close to those predicted by Frank, Gammel, and Watson from the knowledge of the pion-nucleon interaction.

CAPTURE OF NEGATIVE MUONS BY NUCLEI. J. C. Sens,* The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois (Received September 8, 1958).

An experiment conducted to obtain precise values for the rates at which negative muons are captured by nuclei is described. These capture rates were deduced from muon disappearance rates measured by determining the time distribution of electrons resulting from the decay of muons in their lowest atomic orbit. 30 elements were investigated, using scintillation counters as detectors. The data are compared with the general theory of Primakoff and the specific predictions of Tolhoek and Luyten for nuclei with $20 \leq Z \leq 28$. Primakoff's predictions for the effect of the Pauli principle are well borne out by this experiment, and the inferred capture rate of muons by protons is in good agreement with the hypothesis of a Universal Fermi Interaction. New values of effective nuclear charge densities (analogous to Wheeler's Z_{eff}^4) were computed for analyzing the data, using a recent muon mass and up-to-date experimental information on charge distributions (from electron scattering and mesonic x-ray work). These new values are presented in tabular form.

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RADIATIVE MESON-NUCLEON SCATTERING II. R. E. Cutkosky, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received September 11, 1958).

A discussion is given of the corrections which must be made to the results of a previous calculation of radiative meson-nucleon scattering, in which the fixed-source model was used. Curves are given which show the spectrum and angular distribution of gamma rays emitted when positive mesons of energies 130 Mev, 175 Mev, and 220 Mev are scattered by protons.

OPTICAL-MODEL ANALYSIS OF ELASTIC SCATTERING OF 125-Mev K^+ MESONS IN NUCLEAR EMULSIONS. M. A. Melkanoff, O. R. Price, D. H. Stork, and H. K. Ticho, University of California, Los Angeles, California (Received September 19, 1958).

The diffuse-surface optical model has been used to analyze the elastic scattering and interaction cross-section data obtained from 94 meters of positive K-meson track in nuclear emulsion between energies of 100 and 150 Mev. The analysis was carried out on the SWAC digital computer through the use of the existing proton code which required but slight modification, such as the use of the Klein-Gordon equation. A thorough investigation was made of the number of energy intervals and representative emulsion nuclei that were required for adequate accuracy

in the computed averaged cross sections. This number was found to be one energy and two representative emulsion nuclei (heavy and light). The four parameters (R_0, the radius parameter; a, the edge thickness; V, the real part of the potential; and W, the imaginary part of the potential) were varied and the goodness of fit was tested by means of the χ^2 probability, there being nine degrees of freedom. An attractive real potential is fairly clearly ruled out and good fits were obtained for a repulsive real potential for a fairly wide range of physically acceptable geometrical parameters. If the R_0 and a are chosen from electron-scattering experiments, then $V = 21 \pm 4$ Mev and $W = -11.0 \pm 1.5$ Mev. If R_0 is increased to 1.20×10^{-13} cm, then $V = 14 \pm 2.5$ Mev and $W = -7 \pm 1$ Mev.

NUCLEON-ANTINUCLEON SCATTERING. James S. Ball and Jose R. Fulco, Radiation Laboratory, University of California, Berkeley, California (Received September 19, 1958).

By use of the model of the nucleon-antinucleon interaction proposed by Ball and Chew, a calculation of the complex phase shifts at 50 and 260 Mev has been made. The values of annihilation, elastic-scattering, and charge-exchange cross sections, and the angular distributions for $\bar{p}$-p and $\bar{p}$-n elastic scattering are obtained. A comparison with the experimental data shows reasonable agreement. Finally, the parameters of an optical-model potential for antinucleon interaction with complex nuclei are presented.

FURTHER EVIDENCE FOR A VARIATION IN THE RATE OF DENSE EXTENSIVE AIR SHOWERS WITH SOLAR TIME. C. B. A. McCusker, D. E. Page, and R. A. Reid, Dublin Institute for Advanced Studies, Dublin, Ireland (Received September 16, 1958).

The variation in the rate of dense extensive air showers with solar time previously found in Dublin, Ireland, has been confirmed using a similar apparatus situated in Mona, Jamaica. It is suggested that this and other variations in rate with solar time may possibly be due to a periodic change in the structure function of the showers.

STRUCTURE OF THE NEUTRON. Katsumi Tanaka, Argonne National Laboratory, Lemont, Illinois (Received September 10, 1958).

In an earlier paper, the contribution of the meson current to the electromagnetic structure of the nucleon was calculated by use of the dispersion relations for pion-nucleon scattering. Here the contribution of the nucleon current to the electromagnetic structure of the neutron is obtained analogously by use of the dispersion relations for neutron-proton scattering. Only the contribution from the one-pion and two-pion states of nucleon-nucleon scattering is considered. It is found that both states reduce the neutron-electron potential, i.e., lead to a smaller charge radius of the neutron. The contribution from the deuteron state is shown to be negligible. The resulting mean square radii of the neutron, including the contributions of both meson and nucleon current, are $-(0.23 \times 10^{-13}\ \text{cm})^2$ for the charge distribution and $(0.41 \times 10^{-13}\ \text{cm})^2$ for the magnetic moment distribution with $f^2 = 0.08$.

UNIVERSAL WEAK BOSON AND FERMION INTERACTION. Masao Sugawara, Purdue University, Lafayette, Indiana (Received September 29, 1958).

A modified interpretation of an idea of divergenceless currents due to Feynman and Gell-Mann is investigated in detail in which some pairs of bosons (pions and K mesons) are assumed to exist, besides fermion pairs, among members exerting a universal weak interaction. Especially we propose a pair of a neutral K meson and a charged pion, besides a pion pair and pairs of leptons, nucleons, and some hyperons. A strong evidence for this pion-K-meson pair is its surprising fit to short-lived decay of neutral K mesons. An essential presumption is here that pion-muon decay is consistent with the present scheme; its ratio to pion-electron decay is left unchanged. If we further presume a global symmetry of pion coupling to hyperons and K-meson interaction is simply assumed as comparatively weaker, our present scheme seems consistent with all known data on decays of strange particles as far as reliable estimation could have been done. A complete scheme of a universal weak interaction is not obtained uniquely, since we have no particular necessity nor objection to some pairs

of particles, from available theoretical and experimental information. Some decay modes which are yet unobserved but are expected according to the present scheme are summarized in the final section.

G-CONJUGATION AND THE GROUP-SPACE OF THE PROPER LORENTZ GROUP. E. J. Schremp, Nucleonics Division, United States Naval Research Laboratory, Washington, D. C. (Received March 31, 1958; revised manuscript received June 16, 1958).

An earlier proposal of the author for the incorporation of isotopic spin into the foundations of the theory of spin 1/2 particles, based upon a consideration of the group-space of the proper Lorentz group, is here shown to require a certain definite conception of the essentially spatiotemporal character of all "internal" elementary particle phenomena such as isotopic spin. It is further shown that, according to this proposal, the particular spatiotemporal character of the 3-parameter group of isotopic spin rotations of a strongly interacting charge doublet is identical with that of another 3- parameter group of spinor transformations recently applied by Pauli to the neutrino. It thus follows that for leptons, in general, there should exist an analog of isotopic spin rotations, which must be expected to differ from the latter, however, in its physical interpretation. Through a similar generalization of the notion of G-conjugation, a fundamental criterion is then shown to be available for the classification of all spin 1/2 particles into the two families of leptons and baryons, respectively.

QUANTUM THEORY OF GRAVITATION: GENERAL FORMULATION AND LINEARIZED THEORY. R. Arnowitt, Department of Physics, Syracuse University, Syracuse, New York, and S. Deser,* Department of Physics, Harvard University, Cambridge, Massachusetts (Received September 11, 1958).

The problem of quantizing general relativity by using the Schwinger action principle is considered. The advantages of this technique are discussed and the general formulation of the action principle using the Palatini Lagrangian is given. The difficulty in quantizing general relativity is due to the constraint equations. Two types of constraints are distinguished: algebraic constraint equations and differential constraint equations. The former may be dealt with trivially in this formalism. The latter arise due to the presence of function type ("gauge") group invariances. In order to eliminate these variables one must make use of the group transformations themselves. Thus in general relativity the transformation from the full set of variables to the independent canonical ones is a coordinate transformation. The linearized theory is treated in detail from this viewpoint and the full theory is briefly discussed.

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RADIATIVE CORRECTIONS FOR NEARLY ELASTIC SCATTERING. E. L. Lomon,* Cornell University, Ithaca, New York (Received September 2, 1958).

The treatment of large infrared contributions to electron scattering is discussed. An expression is given for any energy resolution, yielding a previously conjectured form in the limit of perfect resolution.

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HIGHER ELECTROMAGNETIC CORRECTIONS TO ELECTRON-PROTON SCATTERING. S. D. Drell and S. Fubini, Stanford University, Stanford, California, and CERN, Geneva, Switzerland (Received September 12, 1958).

Higher-order electromagnetic corrections to the electron-proton scattering amplitudes are studied. The scattering amplitude is subjected to a dispersion analysis which permits the e^4 contribution to be written as the sum of two terms. The first corresponds to radiative corrections to the form factors, the second to virtual photon Compton scattering by the proton. A simple model is constructed for the resonant contributions to Compton scattering which is shown to correct the form-factor analysis negligibly up to ~1 Bev for all scattering angles.